On higher dimensional black holes

Piotr T. Chruściel

Oxford and Tours

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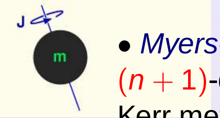
Higher dimensions, vacuum
Myers-Perry, Emparan-Reall



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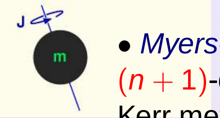

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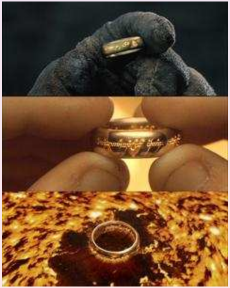

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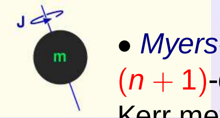

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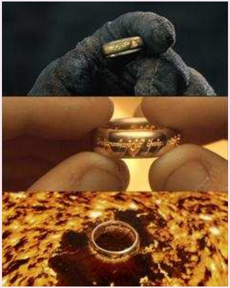

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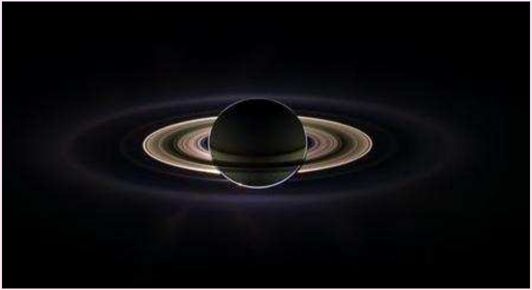

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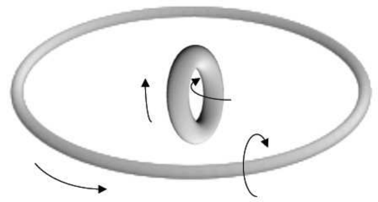

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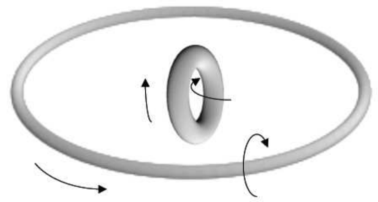

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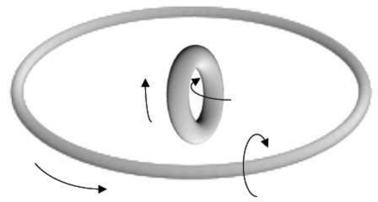

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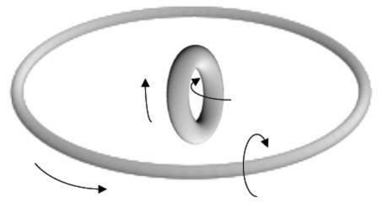

$$
\begin{aligned}
g &= -\frac{F(x)}{F(z)}\left(dt + \sqrt{\frac{\nu}{\xi_F}}\frac{\xi_1 - z}{A}d\psi\right)^2 \\
&\quad + \frac{F(z)}{A^2(x-z)^2}\left[-F(x)\left(\frac{dz^2}{G(z)} + \frac{G(z)}{F(z)}d\psi^2\right)\right. \\
&\quad \left. + F(z)\left(\frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\varphi^2\right)\right] ,
\end{aligned}
$$

where $A > 0$, $\nu > 0$ et ξ_F are constants, and

$$
F(\xi) = 1 - \frac{\xi}{\xi_F} ,
$$

$$
G(\xi) = \nu\xi^3 - \xi^2 + 1 = \nu(\xi - \xi_1)(\xi - \xi_2)(\xi - \xi_3) ,
$$

are polynomials, with ν chosen so that $\xi_1 < 0 < \xi_2 < \xi_3$).

Emparan-Reall: the metric

Five dimensions, vacuum, toroidal *Killing horizon*

The study of the rotation axes at $x = \xi_1$ and $x = \xi_2$ leads to the determination of ξ_F as:

$$\xi_F = \frac{\xi_1 \xi_2 - \xi_3^2}{\xi_1 + \xi_2 - 2\xi_3} \in (\xi_2, \xi_3)\,. \tag{1}$$

Emparan and Reall show

- asymptotic flatness
- existence of a toroidal Killing horizon
- event horizon?
- maximal extensions?

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The event horizon has $\mathbb{R}\times\mathbb{T}^3$ topology

PTC, Cortier, arXiv:0807.2309 [gr-qc]

- Consider the matrix of scalar products of Killing vectors $\{X_i\} = \{\partial_t, \partial_\varphi, \partial_\psi\}$, $i = 1, 2, 3$:

$$h_{ij} = g(X_i, X_j)\ .$$

- if $\det h_{ij} < 0$, some Killing vector has to be timelike
- The event horizon $\mathscr{H}$ is a *null, achronal hypersurface* invariant under isometries, so no Killing vector can be timelike on $\mathscr{H}$
- So on an event horizon $\det h_{ij}$ cannot be negative
- But this determinant *is negative* for $z < \xi_1$ or $z > x_3$

$$\frac{F(x)G(x)F(z)G(z)}{A^4(x-z)^4}\ .$$

- $\{z = \xi_3\}$ is an event horizon: $\xi_F < z < \xi_3$ *must* be within a black hole because z is a time function there, and thus decreases on future directed timelike curves until a singularity $z = \xi_F$ is reached.

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analytic Lorentzian metric on the set

$$\hat{\Omega} := \left\{ \hat{w}, \hat{v} \;\middle|\; -\frac{\xi_3-\xi_1}{\xi_2-\xi_1} \le \hat{v}\hat{w} < \frac{\xi_3-\xi_F}{\xi_F-\xi_2} \right\} \times S^1_{\hat{\psi}} \times S^2_{(x,\varphi)} \; ,$$

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Maximality? Uniqueness of extensions?

Theorem (Kobayashi-Nomizu)

Simply connected, *geodesically complete*, analytic *extensions of semi-Riemannian manifolds are unique*.

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not very useful ...

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PTC, Cortier, arXiv:0807.2309 [gr-qc]

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A pseudo-Riemannian manifold is called s-complete if for every maximally extended geodesic γ that is not complete there exists an obstruction scalar that is *unbounded* on γ.

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Conditions are sharp

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- as well as the possibility of causality violations outside of the black hole region (PS; BS: mentioned but not answered)

Pomeransky-Senkov; Black Saturns

work in progress with J. Cortier, A. Garcia-Parrado (PS), and with M. Eckstein and S. Szybka (BS)

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4. *(PS) Construction of* candidate *maximal* *analytic extensions*

The nondegenerate, connected classification in dimension $4+1$ with three commuting Killing vectors?

Hollands, Yazadjiev (arXiv:0707.2775)

Stationary, vacuum,
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Hollands, Yazadjiev (arXiv:0707.2775)

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• *Hollands Ishibashi Wald*: second Killing vector Y near the horizon; not enough Killing vectors!

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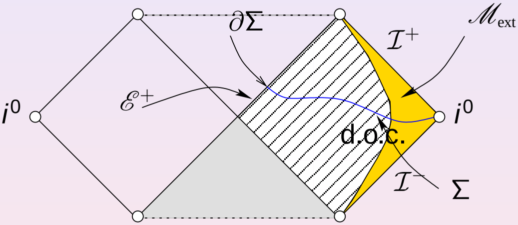

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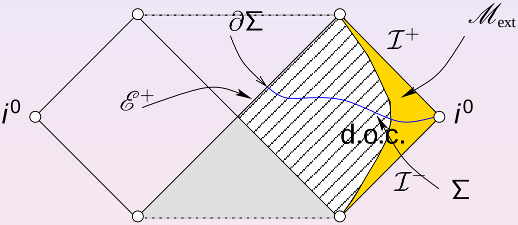

- *Robinson's identity*

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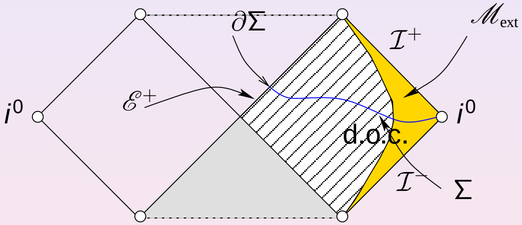

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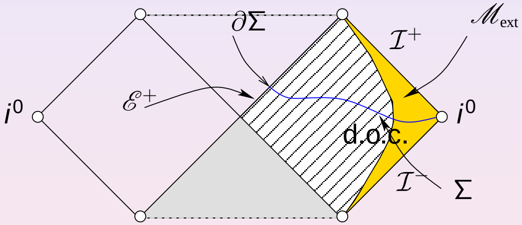

- *Mazur; Breitenlohner Gibbons Maison; Shiromizu Ida; Hollands, Yazadjiev*: a divergence identity for the associated harmonic map implies (?) uniqueness of solutions *open problem:* boundary conditions at the intersection of the axis with the horizon?

Dimension $4+1$: the quotient manifold

$\mathbb{R}^5$ divided by three commuting Killing vectors

- Toy model: $\mathbb{R}^5$ divided by a time translation and two rotations in two orthogonal planes
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- The quotient of $\mathbb{R}^2$ by a rotation is $[0,\infty)$:

$$(x,y) = (\rho\cos\varphi, \rho\sin\varphi)\,,$$

  and the quotient space can be parameterised by ρ
- So the *quotient* space of $\mathbb{R}^4$ by two rotations is $[0,\infty)\times[0,\infty)$, which is a quadrant in $\mathbb{R}^2$, same for the quotient of $\mathbb{R}^5$ by two rotations and a time translation
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- Simple connectedness theorem $\Longrightarrow$ no discrete isotropies (good news...)
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$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Dimension $4+1$: the geometry of the quotient manifold

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Summarising, solutions classified ? by:

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Let M be a *three sphere* with a finite (at least two) number of *balls removed*

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