Geometric Mechanics

Homework 2

Due on October 8

1. Let M and N be differentiable manifolds, $f : M \to N$ and $g : N \to \mathbb{R}$ differentiable maps, and S and T tensor fields on N. Show that:

   (a) $f^*dg = d(g \circ f)$;

   (b) $f^*(T \otimes S) = (f^*T) \otimes (f^*S)$.

2. Consider the usual local coordinates (θ, φ) on the sphere $S^2 \subset \mathbb{R}^3$, defined by the parameterization $\phi :]0, \pi[\times]-\pi, \pi[\to \mathbb{R}^3$ given by

$$\phi(\theta, \varphi) = (\operatorname{sen}\theta \cos\varphi, \operatorname{sen}\theta \operatorname{sen}\varphi, \cos\theta).$$

   (a) Determine the expression, in these coordinates, of the round metric (that is, the Riemannian metric induced in S^2 by the usual Euclidean metric in $\mathbb{R}^3$).

   (b) Compute the length $l(c)$ of the curve $c : \left[\frac{\pi}{4}, \frac{3\pi}{4}\right] \to S^2$ given in local coordinates by $(\theta(t), \varphi(t)) = (t, 0)$.

   (c) Prove that if γ is any other curve on S^2 connecting the points $c\left(\frac{\pi}{4}\right)$ and $c\left(\frac{3\pi}{4}\right)$ then $l(\gamma) \geq l(c)$.