

Geometric Mechanics

Homework 1

Due on September 28

1. Consider the vector fields $X, Y \in \mathfrak{X}(\mathbb{R}^2)$ given by

$$X = x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} \quad \text{and} \quad Y = -y \frac{\partial}{\partial x} + x \frac{\partial}{\partial y}.$$

- (a) Compute $[X, Y]$.
- (b) Determine the flows of X and Y and show that they commute.
- (c) Express X and Y in polar coordinates (r, θ) , defined by the coordinate transformation

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases},$$

and solve (a) and (b) using these coordinates.

2. Determine a vector field $X \in \mathfrak{X}(T\mathbb{R})$ whose flow gives the solutions of the equation of motion for the 1-dimensional harmonic oscillator,

$$\ddot{x} + x = 0.$$