Geometric Mechanics

Homework 9

Due on November 23

1. Consider the symplectic structure on

$$S^2 = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$$

determined by the usual volume form. Compute the Hamiltonian flow generated by the function $H(x, y, z) = z$.

2. Let (M, ω) be a symplectic manifold. Show that:

   (a) $\omega = \sum_{i=1}^{n} dp_i \wedge dx^i$ if and only if $\{x^i, x^j\} = \{p_i, p_j\} = 0$ and $\{p_i, x^j\} = \delta_{ij}$ for $i, j = 1, \ldots, n$;

   (b) M is orientable;

   (c) If M is compact then ω cannot be exact;

   (d) The only sphere that admits a symplectic structure is S^2.