Geometric Mechanics

Homework 7

Due on November 9

1. Let M be an n-dimensional manifold. Recall that the canonical symplectic form $\omega \in \Omega^2(T^*M)$ is given in the standard local coordinates $(x^1, \ldots, x^n, p_1, \ldots, p_n)$ by

$$\omega = dp_1 \wedge dx^1 + \cdots + dp_n \wedge dx^n.$$

   Show that:

   (a) ω is closed ($d\omega = 0$);

   (b) ω is non-degenerate ($\iota(v)\,\omega = 0 \Rightarrow v = 0$);

   (c) $\omega^n = \omega \wedge \cdots \wedge \omega$ is a volume form (in particular T^*M is always orientable, even if M itself is not).

2. Let $(x^1, \ldots, x^n, p_1, \ldots, p_n)$ be the usual local coordinates on T^*M. Compute X_{x^i}, X_{p_i}, $\{x^i, x^j\}$, $\{p_i, p_j\}$ and $\{p_i, x^j\}$.

3. Let $(M, \langle \cdot, \cdot \rangle)$ be a compact Riemannian manifold. Show that for each normal ball $B \subset M$ and each $T > 0$ there exist geodesics $c : \mathbb{R} \to M$ with $\|\dot{c}(t)\| = 1$ such that $c(0) \in B$ and $c(t) \in B$ for some $t \geq T$.