Algebraic and Geometric Methods in Engineering and Physics

2020/2021

Practice Exam 1

Duration: 2 hours

(4/20) **1.** Which of the following sets of matrices form groups under matrix multiplication? Provide either a short proof or a counter-example.

(a) $M_{2\times 2}(\mathbb{Z})$;
(b) $\{A \in M_{2\times 2}(\mathbb{Z}) : \det A \neq 0\}$;
(c) $\{A \in M_{2\times 2}(\mathbb{Z}) : \det A = 1\}$;
(d) $\{A \in M_{2\times 2}(\mathbb{Z}_3) : \det A \neq 0\}$.

(4/20) **2.** Compute a decryption exponent d for the RSA algorithm when the public key is $N = 35$ and the encryption exponent is $e = 7$.

(4/20) **3.** Determine the total number of (equivalence classes of) irreducible representations of the symmetric group S_4, as well as their dimensions.

(4/20) **4.** Prove that the family

$$\tau = \{\varnothing, \mathbb{R}\} \cup \{(-\infty, a) : a \in \mathbb{R}\}$$

defines a topology in $\mathbb{R}$. Which are the compact subsets of $\mathbb{R}$ in this topology?

(4/20) **5.** Determine the weights of the representation of highest weight $(1, 0)$ of the complex semisimple Lie algebra $\mathfrak{so}_5(\mathbb{C})$, whose Cartan matrix is

$$A = \begin{pmatrix} 2 & -2 \\ -1 & 2 \end{pmatrix}.$$