Lie Groups and Lie Algebras

2012/2013

1^{st} Test - 25 January 2013 - 9:00

(4/20) **1.** Show that $\mathfrak{so}(4)$ is isomorphic to $\mathfrak{so}(3) \times \mathfrak{so}(3)$. Is $SO(4)$ isomorphic to $SO(3) \times SO(3)$?

(4/20) **2.** Show that $\exp : \mathfrak{u}(n) \to U(n)$ is surjective but not injective.

3. Recall the definition of a connected Lie subgroup of a matrix Lie group. Show that:

(4/20) (a) All connected Lie subgroups of $SU(2)$ are closed;

(4/20) (b) Not all connected Lie subgroups of $SU(3)$ are closed.

(4/20) **4.** Show that the standard representation of $SO(3)$ is equivalent to the adjoint representation.

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2^{nd} Test - 25 January 2013 - 9:00

(4/20) **1.** Use the Weyl dimension formula for the dimension of a representation with highest weight μ,

$$\dim(\pi) = \frac{\Pi_{\alpha\in R^+}\langle\alpha, \mu+\delta\rangle}{\Pi_{\alpha\in R^+}\langle\alpha,\delta\rangle}$$

(where R^+ is the set of positive roots and δ is half the sum of the positive roots) to obtain the dimension of a representation of $\mathfrak{so}(5,\mathbb{C})$ with highest weight $\mu = m_1\mu_1 + m_2\mu_2$ (where μ_1, μ_2 are the fundamental weights):

$$\dim(\pi) = \frac{1}{6}(m_1+1)(m_2+1)(m_1+m_2+2)(m_1+2m_2+3).$$

2. Show that the following representations of $\mathfrak{so}(5,\mathbb{C})$ are irreducible, and determine their highest weight:

(4/20) (a) The standard representation;

(4/20) (b) The adjoint representation.

(4/20) **3.** (a) Show that $\mathfrak{so}(5,\mathbb{C}) \cong \mathfrak{sp}(2,\mathbb{C})$.

(4/20) (b) What is the irreducible representation of $\mathfrak{so}(5,\mathbb{C})$ with highest weight μ_1?