

Lie Groups and Lie Algebras

Homework 8

Due on November 26

1. Show that:

- (a) The Lie algebra \mathfrak{n} of the Lie group N formed by the upper triangular complex $n \times n$ matrices with 1's on the diagonal is nilpotent.
- (b) The Lie algebra \mathfrak{b} of the Lie group B formed by the upper triangular complex $n \times n$ matrices is soluble.
- (c) $\mathfrak{su}(2)$ is simple.
- (d) $\mathfrak{u}(2)$ is not simple.
- (e) $\mathfrak{u}(2) \cong \mathbb{R} \oplus \mathfrak{su}(2)$.

2. Let \mathfrak{g} be a Lie algebra over some field \mathbb{F} . The **center** of \mathfrak{g} is

$$z(\mathfrak{g}) = \{x \in \mathfrak{g} \mid [x, y] = 0 \text{ for all } y \in \mathfrak{g}\}.$$

Show that:

- (a) $z(\mathfrak{g})$ is an abelian ideal of \mathfrak{g} .
- (b) $\mathfrak{g}/z(\mathfrak{g})$ has a **faithful** (i.e. injective) representation.