

Lie Groups and Lie Algebras

Homework 10

Due on December 10

1. Write the Cartan matrices determined by the following Dynkin diagrams:

- (a) A_l ($l \geq 1$);
- (b) B_l ($l \geq 2$);
- (c) C_l ($l \geq 3$);
- (d) D_l ($l \geq 4$);
- (e) G_2 ;
- (f) F_4 ;
- (g) E_l ($l = 6, 7, 8$).

2. Recall that an **algebra** A over a field \mathbb{F} is a vector space over \mathbb{F} equipped with a bilinear map $A \times A \ni (a, b) \mapsto ab \in A$. A **derivation** in A is a linear map $D : A \rightarrow A$ such that

$$D(ab) = (Da)b + aDb.$$

Show that the set $\text{Der}(A)$ of derivations of A is a Lie algebra over \mathbb{F} (where the Lie bracket is the commutator).