Integrais e Cálculo Vectorial

1. Integrais

1. **Múltiplos:**

$$\iiint_V f(x,y,z)\,dx\,dy\,dz = \iiint_U f(\mathbf{g}(u,v,w))\,|\det D\mathbf{g}|\,du\,dv\,dw$$

2. **De linha:**

(i) Campos escalares:

$$\int_C f = \int_a^b f(\mathbf{g}(t)) \left\| \frac{d\mathbf{g}}{dt}(t) \right\| dt$$

(ii) Campos vectoriais:

$$\int_C \langle \mathbf{F}, d\mathbf{g} \rangle = \int_a^b \left\langle \mathbf{F}(\mathbf{g}(t)), \frac{d\mathbf{g}}{dt}(t) \right\rangle dt$$

3. **De superfície:**

(i) Campos escalares:

$$\iint_S f = \iint_U f(\mathbf{g}(u,v)) \left\| \frac{\partial \mathbf{g}}{\partial u} \times \frac{\partial \mathbf{g}}{\partial v} \right\| du\,dv$$
$$= \iint_U f(\mathbf{g}(u,v)) \sqrt{\det(D\mathbf{g}^t D\mathbf{g})}\,du\,dv$$

(ii) Campos vectoriais:

$$\iint_S \langle \mathbf{F}, \mathbf{n} \rangle = \iint_U \left\langle \mathbf{F}(\mathbf{g}(u,v)), \frac{\partial \mathbf{g}}{\partial u} \times \frac{\partial \mathbf{g}}{\partial v} \right\rangle du\,dv$$

2. Teoremas

1. **Teorema Fundamental do Cálculo para Integrais de Linha:**

$$\int_C \langle \operatorname{grad} \Phi, d\mathbf{g} \rangle = \Phi(\mathbf{g}(b)) - \Phi(\mathbf{g}(a))$$

2. **Teorema de Green:**

$$\iint_U \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy = \oint_{\partial U} P dx + Q dy \equiv \oint_{\partial U} \langle \mathbf{F}, d\mathbf{g} \rangle$$

3. **Teorema da Divergência:**

$$\iiint_U \operatorname{div} \mathbf{F} = \oiint_{\partial U} \langle \mathbf{F}, \mathbf{n} \rangle$$

4. **Teorema de Stokes:**

$$\iint_S \langle \operatorname{rot} \mathbf{F}, \mathbf{n} \rangle = \oint_{\partial S} \langle \mathbf{F}, \mathbf{dg} \rangle$$

5. $\operatorname{rot} \operatorname{grad} \phi = \mathbf{0}$
6. $\operatorname{div} \operatorname{rot} \mathbf{F} = 0$
7. $\operatorname{rot} \mathbf{F} = \mathbf{0}$ + domínio simplesmente conexo $\Rightarrow \mathbf{F} = \operatorname{grad} \phi$
8. $\operatorname{div} \mathbf{F} = 0$ + domínio em estrela $\Rightarrow \mathbf{F} = \operatorname{rot} \mathbf{A}$