Instituto Superior Técnico
Departamento de Matemática

Análise Complexa e Equações Diferenciais

Respostas à Ficha de Trabalho 13

1. (a) $\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin((2n+1)\pi x)}{2n+1}$.
   (b) $\frac{1}{\pi}+\frac{1}{2}\sin x+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{1-4n^2}\cos(2nx)$.
   (c) $\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin\big((2n-1)\pi x\big)=\begin{cases}1 & \text{se} \quad 0<x<1\\ 0 & \text{se} \quad x=0 \text{ ou } x=1\end{cases}$.
   (d) $\frac{1}{2}-\sum_{n=1}^{\infty}\frac{\sin(2n\pi x)}{n\pi}$.
   (e) $\frac{1}{2}-\frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos((2n-1)\pi x)$.

2. (a) $L=\frac{\pi^2}{4}$, $x=0$.
   (b) $L=\frac{\pi}{2}$, $x=0$.

3. $e^{-\alpha^2 t}\operatorname{sen}x-2e^{-25\alpha^2 t}\operatorname{sen}(5x)$.

4. $e^{-(1+\frac{9\pi^2}{L^2})t}\cos\frac{3\pi x}{L}$.

5. (b) $u(x)=T_1+\frac{T_2-T_1}{L}x$;
   (c) $20+40x+\sum_{n=1}^{\infty}\frac{10}{n\pi}\left(3(-1)^n+11\right)e^{-n^2\pi^2\alpha t}\operatorname{sen}(n\pi x)$.

6. (a) $u(x,t)=\operatorname{sen}x+\sum_{n=1}^{\infty}c_n e^{(1-n^2\pi^2)t}\operatorname{sen}(n\pi x)$ com $c_n\in\mathbb{R}$;
   (b) $3e^{(1-4\pi^2)t}\operatorname{sen}(2\pi x)-7e^{(1-16\pi^2)t}\operatorname{sen}(4\pi x)+\operatorname{sen}x$.

7. (a) $\frac{2}{\pi}-\frac{2}{\pi}\sum_{n=2}^{\infty}\frac{1+(-1)^n}{n^2-1}\cos(nx)=\frac{2}{\pi}\left(1-\sum_{n=1}^{\infty}\frac{2}{4n^2-1}\cos(2nx)\right)$;
   (b) $|\operatorname{sen}x|$ M (c) $\frac{2}{\pi}e^{2t}-\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{2e^{(2-4n^2)t}}{4n^2-1}\cos(2nx)$.

8. $\sum_{n=1}^{\infty}\frac{2(1-(-1)^n)}{n^2\pi^2 c}\operatorname{sen}(n\pi ct)\operatorname{sen}(n\pi x)$.

9. $C+\frac{\operatorname{ch}(2\pi x)\,\cos(2\pi y)+\operatorname{ch}(2\pi y)\,\cos(2\pi x)}{2\pi\,\operatorname{sh}(2\pi)}$, $C\in\mathbb{R}$.

10. (a) $\sum_{n=1}^{\infty}B_n\,e^{-(1+n^2)t}\operatorname{sen}(nx)$;
    (b) $\sum_{n=1}^{\infty}\frac{4(1-(-1)^n)}{n^3\pi}e^{-(1+n^2)t}\operatorname{sen}(nx)=\frac{8}{\pi}\sum_{n=0}^{\infty}\frac{e^{-\left(1+(2n+1)^2\right)t}}{(2n+1)^3}\operatorname{sen}\left((2n+1)x\right)$.

11. (a) $\pi+\sum_{n=1}^{\infty}\frac{4((-1)^n-1)}{n^2\pi}\cos\left(\frac{n}{2}x\right)=\pi-\frac{8}{\pi}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\cos\left[\left(n+\frac{1}{2}\right)x\right]$;
    (b) $\pi e^{-\frac{t^2}{2}}+\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n-1}{n^2}e^{-\frac{n^2t+2t^2}{4}}\cos\left(\frac{n}{2}x\right)$.

12. $x+\frac{1}{\sqrt{2}\pi}\operatorname{sen}(2\sqrt{2}\pi t)\operatorname{sen}(2\pi x)\operatorname{sen}(2\pi y)$.