Instituto Superior Técnico
Departamento de Matemática

Análise Complexa e Equações Diferenciais

Respostas à Ficha de Trabalho 12

1. (i) $\begin{bmatrix} e^{-t} & 0 & 0 \\ 0 & e^{2t} & 0 \\ 0 & 0 & e^{-\pi t} \end{bmatrix}$;

(ii) $\begin{bmatrix} e^{2t} & te^{2t} & 0 \\ 0 & e^{2t} & 0 \\ 0 & 0 & e^{-t} \end{bmatrix}$;

(iii) $e^{3t}\begin{bmatrix} 1 & t & t^2/2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix}$;

(iv) $\frac{1}{2}\begin{bmatrix} 3e^{4t}-e^{2t} & -3e^{4t}+3e^{2t} \\ e^{4t}-e^{2t} & -e^{4t}+3e^{2t} \end{bmatrix}$;

(v) $e^{3t}\begin{bmatrix} \cos(2t)+\operatorname{sen}(2t) & -2\operatorname{sen}(2t) \\ \operatorname{sen}(2t) & \cos(2t)-\operatorname{sen}(2t) \end{bmatrix}$;

(vi) $e^{4t}\begin{bmatrix} t+1 & -t & 0 \\ t & 1-t & 0 \\ 0 & 0 & 1 \end{bmatrix}$;

(vii) $\begin{bmatrix} \cos t+\operatorname{sen} t & 2\operatorname{sen} t & 0 \\ -\operatorname{sen} t & \cos t-\operatorname{sen} t & 0 \\ \operatorname{sen} t & \cos t-\operatorname{sen} t-e^{-t} & e^{-t} \end{bmatrix}$;

(viii) $\begin{bmatrix} e^{\pi t} & 0 & 0 & 0 \\ \sqrt{5}te^{\pi t} & e^{\pi t} & 0 & 0 \\ 0 & 0 & e^{t} & \sqrt{2}te^{t} \\ 0 & 0 & 0 & e^{t} \end{bmatrix}$.

2. $\begin{bmatrix} \frac{1}{2}+c_1e^t\cos t+c_2e^t\operatorname{sen} t \\ \frac{3}{2}+c_1e^t\operatorname{sen} t-c_2e^t\cos t \end{bmatrix}$.

3. $x(t)=-1,\ y(t)=-1,\ z(t)=-1+2e^{\frac{t^2}{2}}$.

4. (i) $y(t)=c_1e^{3t}+c_2e^{-t}-\frac{1}{10}\Big(\operatorname{sen} t+2\cos t\Big)$ com $c_1,c_2\in\mathbb{R}$;

(ii) $y(t)=\Big(c_1+c_2t+\frac{t^3}{6}\Big)e^t$ com $c_1,c_2\in\mathbb{R}$;

(iii) $y(t)=\cos(\sqrt{2}t)\Big(c_1e^{\sqrt{2}t/2}+c_2e^{-\sqrt{2}t/2}\Big)+\operatorname{sen}(\sqrt{2}t)\Big(c_3e^{\sqrt{2}t/2}+c_4e^{-\sqrt{2}t/2}\Big)$
$+\frac{e^{2t}}{102}\Big(-4\cos t-\sin t\Big)+t$ com $c_1,c_2,c_3,c_4\in\mathbb{R}$;

(iv) $y(t) = c_1 + c_2 t + c_3 e^{2t} - \frac{1}{8}t^2 - \frac{1}{12}t^3$ com $c_1, c_2, c_3 \in \mathbb{R}$.

5. (i) $y(t) = 5 - 5e^t + 3te^t + 2t$; (ii) $y(t) = 8 - 8e^t + 4te^t + 4t + \frac{t^2}{2}$;

   (iii) $y(t) = 4 - 4e^t + 2te^t + 2t + \frac{t^2}{2}e^t$.

6. (i) $\begin{bmatrix} \frac{1}{3}\Big(3c_1e^{-t} + 3c_2e^{2t} - 10e^{2t} - 5te^{2t}\Big) \\ \frac{1}{3}\Big(-3c_1e^{-t} + 6c_2e^{2t} - 10te^{2t} - 10e^{2t}\Big) \end{bmatrix}$ com $c_1, c_2 \in \mathbb{R}$;

   (ii) $\begin{bmatrix} c_1e^{2t} + c_2e^{-2t} - \frac{8}{12}e^t - \frac{1}{4} \\ c_1e^{2t} - 3c_2e^{-2t} - e^t - \frac{3}{4} \end{bmatrix}$ com $c_1, c_2 \in \mathbb{R}$.

7. $\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} e^{\sqrt{2}t}(t^2 + 2t) \\ e^{\sqrt{2}t}(t+1) \\ e^{-t}(t+1) \end{bmatrix}$.

8. $y(t) = \frac{e^{-2t}}{\pi} \begin{bmatrix} 0 \\ 0 \\ -\operatorname{sen}(\pi t) \\ 1 - \cos(\pi t) \end{bmatrix}$.

9. (a) $y(t) = c_1e^t + c_2te^t + e^t(t\log t - t)$ com $c_1, c_2 \in \mathbb{R}$;

   (b) $y(t) = c_1e^{-t} + c_2e^{-2t} - e^{-2t}\operatorname{sen}(e^t)$ com $c_1, c_2 \in \mathbb{R}$.

10. $y(x) = 3e^{-2x} - (5 + 2\log 2)e^{-x} + (e^{-x} + e^{-2x})\log(1 + e^x)$.

11. (a) $\lambda = k = 1$ (b) $y(t) = (2 + \log 2)t - 2e^{t-2} - 1 - t\log t$.

12. (c) $e^t \begin{bmatrix} 1 & 2t & 3t + 2t^2 \\ 0 & 1 & 2t \\ 0 & 0 & 1 \end{bmatrix}$.

13. $\begin{bmatrix} e^{at}\cos(bt) & -e^{at}\operatorname{sen}(bt) \\ e^{at}\operatorname{sen}(bt) & e^{at}\cos(bt) \end{bmatrix}$.

14. (i) 4.56 C; (ii) 0.05 segundos.