Instituto Superior Técnico
Departamento de Matemática

Análise Complexa e Equações Diferenciais

Respostas à Ficha de Trabalho 11

1. (a) $(\alpha, \beta) = (1, 0)$ ou $(\alpha, \beta) = (0, -2)$.

(b) $x(t) = -4 + 2t; \quad y(t) = 2 - \frac{4}{t^2}$ com $t \in]1, +\infty[$.

2. (i) $e^{2t} \begin{bmatrix} (-c_1 + c_2)\,\mathrm{sen}\, t + c_1 \cos t \\ (-2c_1 + c_2)\,\mathrm{sen}\, t + c_2 \cos t \end{bmatrix} \quad c_1, c_2 \in \mathbb{R}$;

(ii) $\begin{bmatrix} \frac{1}{3}\Big(3c_1 e^{-t} + 3c_2 e^{2t}\Big) \\ \frac{1}{3}\Big(-3c_1 e^{-t} + 6c_2 e^{2t}\Big) \end{bmatrix}$ com $c_1, c_2 \in \mathbb{R}$;

(iii) $\begin{bmatrix} (c_1 + c_2 t)e^{2t} \\ (c_1 + c_2 + c_2 t)e^{2t} \end{bmatrix}$ com $c_1, c_2 \in \mathbb{R}$;

(iv) $\begin{bmatrix} c_2 + 3c_3 e^{2t} \\ c_1 e^{-t} - 2c_3 e^{2t} \\ -2c_1 e^{-t} + c_2 + c_3 e^{2t} \end{bmatrix}$ com $c_1, c_2, c_3 \in \mathbb{R}$;

(v) $\begin{bmatrix} c_1 + \frac{c_3}{4} e^{-4t} \\ c_2 e^t + \frac{4}{5} c_3 e^{-4t} \\ c_3 e^{-4t} \end{bmatrix}$ com $c_1, c_2, c_3 \in \mathbb{R}$;

(vi) $\begin{bmatrix} c_1 e^t + c_2 t e^t \\ (2c_3 - c_1 - c_2)e^t + c_2 t e^t \\ c_2 t e^t + c_3 e^t \end{bmatrix}$ com $c_1, c_2, c_3 \in \mathbb{R}$.

3. $Y(t) = \begin{bmatrix} 1 & e^{2t} & 0 \\ 1 & 0 & e^t \\ 0 & 0 & e^t \end{bmatrix}$; solução do problema de valor inicial: $\begin{bmatrix} e^{2t} \\ 0 \\ 0 \end{bmatrix}$.

4. $\mathbf{y}(t) = \begin{bmatrix} \alpha e^{-2t} \\ 3\alpha t e^{-2t} + \beta e^{-2t} + 1 \\ \gamma e^t \cos(2t) + \delta e^t\,\mathrm{sen}(2t) \\ -\gamma e^t\,\mathrm{sen}(2t) + \delta e^t \cos(2t) \end{bmatrix}$ com $\alpha, \beta, \gamma, \delta \in \mathbb{R}$.

5. $\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} \mathrm{sen}\, t + \cos t \\ 2\,\mathrm{sen}\, t \\ 2 - e^{-1+\cos t} \end{bmatrix}$.

6. A equação é equivalente ao sistema de três equações de primeira ordem $\frac{dx}{dt} = \left(x_2, x_3, \frac{\mathrm{sen}(t) x_3 + e^{x_1}}{t^2 + x_1^2}\right)$ que tem solução única pelo Teorema de Picard.

7. (i) $y(t) = c_1 + c_2 t + c_3 e^{2t}$ com $c_1,\ c_2,\ c_3 \in \mathbb{R}$;

(ii) $y(t) = (A + Bt)e^{-2t} + (C + Dt + Et^2)e^t \cos 2t + +(F + Gt + Ht^2)e^t \operatorname{sen} 2t + Ie^t$ com $A, B, C, D, E, F, G, H, I \in \mathbb{R}$;

(iii) $y(t) = c_1 \cos t + c_2 t \cos t + c_3 \operatorname{sen} t + c_4 t \operatorname{sen} t$ com $c_1, c_2, c_3, c_4 \in \mathbb{R}$.

8. (i) $y(t) = \cos t + \operatorname{sen} t$; (ii) $y(t) = 4e^t - 3e^{2t} + 3te^{2t}$; (iii) $y(t) = 0$.

9. (i) $y(t) = c_1 + e^{2t}\Big(c_2 \cos t + c_3 \sin t\Big)$ com $c_1, c_2, c_3 \in \mathbb{R}$;

(ii) $y(0) = \alpha \in \mathbb{R}$, $y'(0) = y''(0) = 0$.