Domain Decomposition methods for Isogeometric Analysis and applications to computational electrocardiology

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Abstract

Isogeometric Analysis (IGA) is a non-standard numerical method for partial differential equations (PDEs), which was introduced in [1] and first analyzed in [2]. The ultimate goal of IGA is to adopt the geometry description from a Computer Aided Design (CAD) parametrization, and use it for the analysis within a PDE solver. Non-uniform rational

B-splines (NURBS) are a standard in CAD community mainly because they are extremely convenient for the representation of free-form surfaces and very efficient algorithms have been developed to evaluate, refine and derefine them. In IGA, those same basis functions that represent the CAD geometry are also used as the basis for the discrete solution space of PDEs, thus following an isoparametric paradigm. IGA methodologies have been studied and applied in fields as diverse as fluid dynamics, structural mechanics and electromagnetics.

Domain decomposition methods are a major area of recent research in numerical analysis for PDEs. They provide robust, parallel and scalable preconditioned iterative methods for the large linear systems arising from the discretizaton of continuous problems.

In this talk, we propose Overlapping Additive Schwarz (OAS) and Balancing Domain Decomposition by Constraints (BDDC) preconditioners for IGA discretizations of elliptic problems, see [3, 4, 5]. We construct the OAS and BDDC preconditioners both in the parametric space and in the physical space, we prove that our proposed method are scalable and we validate the theoretical results by 2D and 3D numerical experiments. Moreover, we apply the OAS and BDDC preconditioners to the solution of the linear systems arising from the IGA approximation of the Bidomain model of electrocardiology, a degenerate system of parabolic PDEs modeling the bioelectrical activity of the cardiac tissue.

Keywords: Isogeometric analysis; Domain Decomposition methods; Overlapping Additive Schwarz preconditioners; BDDC preconditioners.

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