

Motion of aerobic bacteria in liquids

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Abstract

Let us fix a number $T > 0$ and a domain $\Omega \subset \mathbb{R}^d$, with $d = 2, 3$, which can be a bounded open set locally located on one side of its C^2 -smooth boundary $\partial\Omega$ or the whole space \mathbb{R}^d itself. In the cylinder $Q_T = (0, T) \times \Omega$, we consider the following set of equations

$$\partial_t n + u \cdot \nabla n - \Delta(n^m) = -\nabla \cdot (\chi(c)n\nabla c) + f(n), \quad (1)$$

$$\partial_t c + u \cdot \nabla c - \Delta c = -k(c)n, \quad (2)$$

$$\partial_t u + u \cdot \nabla u - \Delta u + \nabla p = -n\nabla\phi, \quad (3)$$

$$\nabla \cdot u = 0. \quad (4)$$

Here $c(t, x) : Q_T \rightarrow \mathbb{R}$, $n(t, x) : Q_T \rightarrow \mathbb{R}$, $u(t, x) : Q_T \rightarrow \mathbb{R}^d$ and $p(t, x) : Q_T \rightarrow \mathbb{R}$ are the oxygen concentration, cell concentration, fluid velocity, and hydrostatic pressure, respectively. The scalar functions k , χ and f determine the oxygen consumption rate, chemotactic sensitivity, and bacterial growth, resp., $\phi : Q_T \rightarrow \mathbb{R}$ is the potential produced by the action of physical forces on the cells, and $m \geq 1$ is the nonlinear diffusion exponent. The cases $m = 1$ and $f \equiv 0$ are not excluded.

The system is complemented with the no-flux boundary conditions for n^m and c , and the no-slip condition for u ,

$$\frac{\partial n^m(t, x)}{\partial \nu} = 0, \quad \frac{\partial c(t, x)}{\partial \nu} = 0, \quad u(t, x) = 0, \quad x \in \partial\Omega, \quad (5)$$

and with the initial conditions

$$n(0, x) = n_0(x), \quad c(0, x) = c_0(x), \quad u(0, x) = u_0(x), \quad x \in \Omega. \quad (6)$$

The model (1)–(4) with $f \equiv 0$ and $m = 1$ was suggested in [5] in order to describe the combination of chemotaxis, metabolism, cell–cell signaling, buoyancy, diffusion, and mixing in drops full of aerobic bacteria. The coupled system consisting of the equations of cell and oxygen balance and of the ones of fluid motion and continuity is based on general considerations from the bioconvection theory (cf. [2, 4]). The nonlinear-diffusion variant

of the model (i.e. when $m > 1$) was proposed in [1]. In this paper, we also admit the proliferation/death term f in the cell balance equation (1). Note that the convectionless chemotactic models taking into account the role of cell kinetics were put forward in [3]. The typical kinetic terms are $f(n) = Kn(1-n)$ (Fisher's one) and $f(n) = Kn(1-n)(n-\alpha)$ (the Allee effect).

We prove existence of global weak solutions to the problem. We distinguish between two cases determined by the cell diffusion term and the space dimension, which are referred to as the supercritical and subcritical ones. At the first case, the choice of the kinetic function enjoys wide range of possibilities: in particular, it can be zero. Our results are new even at the absence of the kinetic term. At the second case, the restrictions on the kinetic function are less relaxed: for instance, it cannot be zero but can be Fisher-like. In the case of linear cell diffusion, the solution is regular and unique provided the domain is the whole plane. In addition, we study the long-time behaviour of the problem, find dissipative estimates, and construct attractors. The talk is mainly based on [6].

Keywords: aerobic bacteria, bioconvection, oxytaxis, trajectory attractor, global solvability, Navier-Stokes, Keller-Segel, reaction-diffusion

References

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