On the regularity, the stability and optimal control of the free boundary in two-phases heterogeneous stationary problems

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Abstract

We extend some properties to the solutions of free boundary problems of obstacle type with two phases for a class of heterogeneous quasilinear elliptic operators. Under a natural non degeneracy assumption on the interface, corresponding to the zero level set, we prove a continuous dependence result for the characteristic functions of each phase and we establish sharp estimates on the variation of its Lebesgue measure with respect to the L1-variation of the data, in a rather general framework. For elliptic quasilinear equations, we show that the characteristic functions of both phases are of bounded variation for non degenerating heterogeneous forces. This extends recent results for the obstacle problem and is a first result on the regularity of the free boundary of the heterogeneous two phases problem and allows to prove the existence of an optimal non degenerated interface with minimal perimeter and closer to a given set.