Optimal nonlinear stability in PDEs and applications

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Abstract

The stability of a basic motion plays a central role in dynamical systems. There are several methods for investigating stability.

The spectral methods study the real parts of the eigenvalues of the linear operator of the system and give sufficient conditions of instability. The Lyapunov second method introduces suitable energy-norms (or Lyapunov functions) and gives sufficient conditions for nonlinear stability of the flow. If the linear operator is symmetric with respect to an energy-norm, the linear instability threshold coincides with the energy nonlinear stability.

Many fluid systems show stabilizing effects mainly due to forces which give skew-symmetric contributions to linear operator (rotation, chemical concentration, magnetic fields). The energy norm is not sensitive to such effects.

Here we prove the coincidence of linear and nonlinear thresholds when the stabilizing effects are present. The coincidence is obtained with the introduction of optimal Lyapunov functions, [1]. Some biomedical applications are given:

a) epidemic model with evolution, [2],

b) application to a model of glia aggregation in the brain, [3].

References

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