Decay of viscoelastic waves with memory

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Abstract

The displacement u of a viscoelastic body under the action of an external force f is given by Newton's second law $\rho \frac{d^2u}{dt^2} = \nabla \cdot \sigma + f$, where ρ is the mass density, $\sigma(t)$ is the stress tensor.

We assume that the stress tensor $\sigma(t)$ and the strain tensor $\epsilon(t)$ are related by the constitutive equation $\sigma(t) = E(0)D\epsilon(t) - \int_0^t \frac{\partial}{\partial s} E(t-s)D\epsilon(s)\, ds$, where D is an elastic tensor, the stress relaxation function E is nonnegative and monotone decreasing and the strain depends on the displacement $\epsilon(t) = \frac{1}{2}\big(\nabla u(t) + \nabla u(t)^t\big)$.

If the viscoelastic behaviour is described by Maxwell-Wiechert model (with only one Maxwell arm), that is $E(t) = E_0 + E_1 e^{-\alpha_1 t}$, where E_0 is the Young modulus of the spring arm, E_1 is the Young modulus of the Maxwell arm and $\alpha_1 = \frac{E_1}{\mu_1}$ being μ_1 the associated viscosity, we obtain the following second order integro-differential equation

$$\rho \frac{d^2u}{dt^2}(t) - D_1 \Delta u(t) = -D_2 \int_0^t K_{er}(t-s) \Delta u(s)\ ds + f, \qquad (1)$$

with $D_1 = D(E(0) + E_1)$, $D_2 = \frac{E_1}{2}$ and $\tau = \alpha_1^{-1}$. Without being exhaustive, we mention [1], [2], [3],[4] and [5] for the study of qualitative properties of partial differential problems defined by equations of type of (1).

In this talk we consider the general wave equation with memory

$$\frac{d^2u}{dt^2}(t) + c\frac{du}{dt}(t) + \mathcal{A}u(t) = \int_0^t k_{ker}(t-s)\mathcal{B}u(s)ds + f(t),\ t \in \mathbb{R}^+, \qquad (2)$$

where k_{er} denotes a positive function, $\mathcal{A}$ and $\mathcal{B}$ are second order differential operators, and we analyse the behavior of several energy functionals under general assumptions. We introduce numerical wave equations that mimic their continuous counterpart and their behaviour will be explored.

Keywords: Wave equation, memory effect, energy estimates, numerical solution.

References

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