Effective models for transport processes through membranes

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Abstract

In this presentation, we develop multiscale methods for the derivation and analysis of effective models in environments containing membranes. At the microscopic level, where membranes are modeled as thin heterogeneous layers, the model consists of nonlinear reaction-diffusion equations within each subdomain. At the macroscopic level, membranes are reduced to interfaces, and effective transmission conditions and/or effective equations at these interfaces are derived. It turns out that the form of the effective laws at the interface depends on the scaling of the microscopic system as well as of the type of microscopic transmission conditions imposed at the bulk-layer interface. For the derivation of macroscopic (effective) models, we first generalize the concept of weak and strong two-scale convergence to flat membranes with periodic structure. This allows to derive cell-problems, which approximate at zeroth order the processes in the membrane. For the derivation of effective transmission conditions at the interfaces, we define test-functions of boundary-layer type, adapted to dimension reduction. In case of curved membranes, we introduce the notions of locally periodic functions on manifolds and thin layers and two-scale convergence with respect to charts.

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Keywords: thin heterogeneous layers, weak and strong two-scale convergence with respect to charts, effective equations at curved membranes.