Lecture 5 suggested problems

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Exercise on C_n: Consider a coupling argument to prove that the lazy, simple random walk on C_n needs at most order n^2 steps to reach stationarity.

Inverse Riffle shuffles: Consider a deck of n card. For each card, we flip a fair coin, independently. If heads, we mark the card with a zero. If tails, we mark it with a one. Once all cards have been marked, we move all the zero cards to the top, keeping their relative order fixed.

(a) Explain why this is a hyperplane arrangement shuffle.

(b) Consider a coupling to bound the mixing time of this shuffle.

Biased random walk on the path: Let X_t be the biased random walk on the path with n nodes, moving to the right with probability $2/3$ and to the left with probability $1/3$. Whenever such a move is impossible, the walk remains fixed. Come up with a coupling argument to provide an upper bound for the total variation distance of order n.

Full binary tree: Consider the full binary tree on with n nodes and depth h.

(a) Come up with a coupling argument that will provide an upper bound for the total variation distance of order n.

(b) Use the eigenvalue/eigenvector exercise of Lecture 4, to prove that there is a constant c such that every non-trivial eigenvalue of the transition matrix satisfies $\lambda \leq 1 - \frac{c}{n}$.