**Exercise 1** – Lie-Kato-Trotter product formula

Prove the Lie-Kato-Trotter product formula

$$e^{i(A+B)}\,\psi = \lim_{N\to\infty}\left(e^{iA/N}\,e^{iB/N}\right)^N\,\psi$$

for the case of self-adjoint bounded operators $A$ and $B$ acting on a Hilbert space $\mathcal{H}$ ($\psi \in \mathcal{H}$).

**Exercise 2** – Path integral formulation of quantum mechanics

Let $\psi \in L^2(\mathbb{R})$. Apply the Trotter product formula to the operator

$$\hat{H} = -\frac{\hbar^2}{2m}\,\frac{d^2}{dx^2} + V(x)$$

to derive the formal expression

$$\left(e^{-it\hat{H}/\hbar}\,\psi\right)(x_0) = \lim_{N\to\infty} C \int_{(\mathbb{R})^N} \exp\left\{\frac{i}{\hbar}\sum_{k=0}^{N-1}\frac{t}{N}\left[\frac{m}{2}\left(\frac{x_{k+1}-x_k}{t/N}\right)^2 - V(x_k)\right]\right\}\psi(x_N)\prod_{k=1}^{N} dx_k\ .$$

Determine $C$.