

Exercise 1 – WS supersymmetry

Show that under an $N = 1$ supersymmetry transformation the WS current

$$G_a = \frac{i}{\sqrt{2\alpha'}} \rho^b \rho_a \psi^\mu \partial_b X_\mu$$

transforms into the WS energy-momentum stress tensor $T_{ab} = T_{ab}^X + T_{ab}^\psi$, i.e. $\delta_\epsilon G_a = -iT_{ab}\rho^b\epsilon$.

Hint: Use the Fierz rearrangement formula for spinors in two dimensions (with $\bar{\rho} \equiv \rho_0\rho_1$)

$$(\bar{\lambda}\psi)\chi_\alpha = -\frac{1}{2} [(\bar{\lambda}\chi)\psi_\alpha + (\bar{\lambda}\bar{\rho}\chi)(\bar{\rho}\psi)_\alpha + (\bar{\lambda}\rho^a\chi)(\rho_a\psi)_\alpha]$$

and the equation of motion for ψ_α^μ .

Exercise 2 – GSO projection: the fermion number $(-1)^F$

Consider the fermion number operator

$$(-1)^{\hat{F}} \equiv \begin{cases} e^{i\pi\hat{F}} & \text{in the NS sector} \\ b_0^1 \cdots b_0^8 e^{i\pi\hat{F}} & \text{in the R sector} \end{cases}$$

where $(i = 1, \dots, 8)$

$$\hat{F} = \begin{cases} \sum_{r \in \mathbb{N}_0 + \frac{1}{2}} b_{-r}^i b_r^i & \text{in the NS sector} \\ \sum_{r \in \mathbb{N}} b_{-r}^i b_r^i & \text{in the R sector} \end{cases}$$

Show that

$$\{b_r^i, (-1)^{\hat{F}}\} = 0$$

for all the modes in the NS sector, and for all the modes in the R sector.

Exercise 3 – RNS open string

Consider the RNS open string with $\sigma \in [0, \pi]$. For the open string, the variation

$$(\psi_- \cdot \delta\psi_- - \psi_+ \cdot \delta\psi_+) \Big|_{\sigma=0}^{\sigma=\pi} = 0$$

has to be cancelled at each of the boundaries separately. NN boundary conditions are defined by

$$\psi_+^\mu(0) = \psi_-^\mu(0) \quad , \quad \psi_+^\mu(\pi) = \eta \psi_-^\mu(\pi) \quad , \quad \eta = \pm 1 \quad , \quad \mu = 0, \dots, D-1 \quad .$$

The choice $\eta = 1$ is called Ramond sector, while the choice $\eta = -1$ is called Neveu-Schwarz sector. Obtain the mode expansion for ψ for both these sectors. You may want to define a suitable extension of ψ_+^μ to the interval $\sigma \in [0, 2\pi]$.