Differential Geometry - 1st Semester 2012/13

Ficha 5 (due Thursday November 29)

1) Let M be a smooth manifold, and $\omega \in \Omega^{k-1}(M)$ a $(k-1)$-form.
Prove Stokes theorem for a smooth singular k-chain c:

$$\int_c d\omega = \int_{\partial c} \omega \ .$$

2) Compute $H^k(M)$ for

a) $M = \mathbb{T}^d$,

b) $M = \mathbb{P}^d$.

3) Consider the long sequence of the *Zigzag Lemma*,

$$\ldots \longrightarrow H^k(A) \longrightarrow^f H^k(B) \longrightarrow^g H^k(C) \longrightarrow^{d^*} H^{k+1}(A) \longrightarrow \ldots$$

Show that $\operatorname{Im} d^* = \ker\ f$ and $\operatorname{Im} g = \ker\ d^*$.

4) Given an exact sequence of vector spaces

$$0 \longrightarrow C^0 \longrightarrow \ldots \longrightarrow C^k \longrightarrow \ldots \longrightarrow C^d \longrightarrow 0 \ ,$$

show that

$$\sum_{i=0}^{d} (-1)^i \, dim\, C^i = 0 \ .$$

5) Prove the *The Five Lemma*.