INSTITUTO SUPERIOR TÉCNICO

Mestrado em Engenharia Electrotécnica e de Computadores

Ano Lectivo: 2007/2008 Semestre: 2º

MATEMÁTICA COMPUTACIONAL

Soluções dos Exercícios Recomendados

[1.5] (a) (b)

$x = 0.314159265358979\ldots \times 10$ $\quad$ $y = 0.314150943396226\ldots \times 10$

$\tilde{x} = 0.314159 \times 10$ $\quad$ $\tilde{y} = 0.314151 \times 10$

$e_{\tilde{x}} = 0.265 \times 10^{-5}$ $\quad$ $e_{\tilde{y}} = -0.566 \times 10^{-6}$

$\delta_{\tilde{x}} = 0.844 \times 10^{-6}$ $\quad$ $\delta_{\tilde{y}} = -0.180 \times 10^{-6}$

(c) (d) (e) Pondo $x \boxed{\circ} y = \text{fl}(\text{fl}(x) \circ \text{fl}(y))$:

$\circ$	$x \circ y$	$x \boxed{\circ} y$	$e_{x \boxed{\circ} y}$	$\delta_{x \boxed{\circ} y}$	$a.s.$
$\times$	$0.986934295891887\ldots \times 10$	0.986934×10	0.296×10^{-5}	0.300×10^{-6}	6
$\div$	$0.100002649033188\ldots \times 10$	0.100003×10	-0.351×10^{-5}	-0.351×10^{-5}	6
$+$	$0.628310208755205\ldots \times 10$	0.628310×10	0.209×10^{-5}	0.332×10^{-6}	6
$-$	$0.832196275290875\ldots \times 10^{-4}$	0.800000×10^{-4}	0.322×10^{-5}	0.387×10^{-1}	1

(f) $x = 0.314159265358979\ldots \times 10$ $\quad$ $y = 0.314150943396226\ldots \times 10$

$\tilde{x} = 0.314159265 \times 10$ $\quad$ $\tilde{y} = 0.314150943 \times 10$

$e_{\tilde{x}} = 0.359 \times 10^{-8}$ $\quad$ $e_{\tilde{y}} = 0.396 \times 10^{-8}$

$\delta_{\tilde{x}} = 0.114 \times 10^{-8}$ $\quad$ $\delta_{\tilde{y}} = 0.126 \times 10^{-8}$

$x - y = 0.832196275290875\ldots \times 10^{-4}$

$x \boxed{-} y = 0.832200000 \times 10^{-4}$

$e_{x \boxed{-} y} = -0.373 \times 10^{-9}$

$\delta_{x \boxed{-} y} = -0.448 \times 10^{-5}$

a.s.: 5

[1.12] (a) Alg. 1: $u_1 = x \times x, \qquad u_2 = y \times y, \qquad z = u_3 = u_1 - u_2$

Alg. 2: $u_1 = x + y, \qquad u_2 = x - y, \qquad z = u_3 = u_1 \times u_2$

Alg. 3: $u_1 = x + y, \qquad u_2 = u_1 \times x, \qquad u_3 = u_1 \times y, \qquad z = u_4 = u_2 - u_3$

Alg. 1: $\delta^L_{\tilde z} = \delta^L_{f(\tilde x,\tilde y)} + \delta^L_{\text{arr}}$

$$\delta^L_{f(\tilde x,\tilde y)} = \frac{2}{x^2 - y^2}\left(x^2\delta_{\tilde x} - y^2\delta_{\tilde y}\right),$$

$$\delta^L_{\text{arr}} = \frac{x^2}{x^2-y^2}\,\delta_{\text{arr},1} - \frac{y^2}{x^2-y^2}\,\delta_{\text{arr},2} + \delta_{\text{arr},3}$$

Alg. 2: $\delta^L_{\tilde z} = \delta^L_{f(\tilde x,\tilde y)} + \delta^L_{\text{arr}}$

$$\delta^L_{f(\tilde x,\tilde y)} = \frac{2}{x^2 - y^2}\left(x^2\delta_{\tilde x} - y^2\delta_{\tilde y}\right),$$

$$\delta^L_{\text{arr}} = \delta_{\text{arr},1} + \delta_{\text{arr},2} + \delta_{\text{arr},3}$$

Alg. 3: $\delta^L_{\tilde z} = \delta^L_{f(\tilde x,\tilde y)} + \delta^L_{\text{arr}}$

$$\delta^L_{f(\tilde x,\tilde y)} = \frac{2}{x^2 - y^2}\left(x^2\delta_{\tilde x} - y^2\delta_{\tilde y}\right),$$

$$\delta^L_{\text{arr}} = \delta_{\text{arr},1} + \frac{x}{x-y}\,\delta_{\text{arr},2} - \frac{y}{x-y}\,\delta_{\text{arr},3} + \delta_{\text{arr},4}$$

(b) Pondo $\delta_{\tilde x} = \delta_{\tilde y} = 0$, $x = r\cos\theta, y = r\sin\theta$ e sendo u a unidade de arredondamento do sistema de ponto flutuante onde se fazem os cálculos obtém-se:

Alg. 1: $\left|\delta^L_{\tilde z}\right| \le K_1(\theta)u \qquad$ Alg. 2: $\left|\delta^L_{\tilde z}\right| \le 3u \qquad$ Alg. 3: $\left|\delta^L_{\tilde z}\right| \le K_3(\theta)u$

$$K_1(\theta) = \frac{x^2+y^2}{|x^2-y^2|} + 1 = \frac{1}{|\cos 2\theta|} + 1$$

$$K_3(\theta) = \frac{|x|+|y|}{|x-y|} + 2 = \frac{|\cos\theta| + |\sin\theta|}{|\cos\theta - \sin\theta|} + 2$$

$$\min\{K_1(\theta), 3, K_3(\theta)\} =$$

$$\begin{cases} K_1(\theta), & |\theta| \le \frac{\pi}{6} \ \vee\ \frac{2\pi}{6} \le |\theta| \le \frac{4\pi}{6} \ \vee\ \frac{5\pi}{6} \le |\theta| \le \pi \\ 3, & \frac{\pi}{6} \le |\theta| \le \frac{2\pi}{6} \ \vee\ \frac{4\pi}{6} \le |\theta| \le \frac{5\pi}{6} \end{cases}$$

[1.15] (a) $S = 0.641371258 \times 10^{-3}$

(b) $\tilde{S}_1 = 0.64137126 \times 10^{-3}$; $\quad \tilde{S}_2 = 0.64100000 \times 10^{-3}$

(c) $\delta_{\tilde{S}_1} = -0.312 \times 10^{-8}$; $\quad \delta_{\tilde{S}_2} = 0.579 \times 10^{-3}$

(d) Alg. 1: $u = a + b, \qquad S_1 = u + c$

$$\delta^L_{\tilde{S}_1} = \delta^L_S + \frac{a+b}{S}\,\delta_{\text{arr},1} + \delta_{\text{arr},2}, \qquad \delta^L_S = \frac{1}{S}\left(a\delta_{\tilde{a}} + b\delta_{\tilde{b}} + c\delta_{\tilde{c}}\right)$$

Alg. 2: $u = b + c, \qquad S_2 = u + a$

$$\delta^L_{\tilde{S}_2} = \delta^L_S + \frac{b+c}{S}\,\delta_{\text{arr},1} + \delta_{\text{arr},2}, \qquad \delta^L_S = \frac{1}{S}\left(a\delta_{\tilde{a}} + b\delta_{\tilde{b}} + c\delta_{\tilde{c}}\right)$$

Alg. 3: $u = c + a, \qquad S_3 = u + b$

$$\delta^L_{\tilde{S}_3} = \delta^L_S + \frac{a+c}{S}\,\delta_{\text{arr},1} + \delta_{\text{arr},2}, \qquad \delta^L_S = \frac{1}{S}\left(a\delta_{\tilde{a}} + b\delta_{\tilde{b}} + c\delta_{\tilde{c}}\right)$$

Devem somar-se primeiro os dois números cuja soma tenha o menor valor. Para os valores de a, b, c da alínea (a) devem pois somar-se primeiro os números a e b.

[1.16] (a) $f(4.71) = -0.14263899 \times 10^2$

(b) $\tilde{f}_1(4.71) = -0.134 \times 10^2, \quad \tilde{f}_2(4.71) = -0.143 \times 10^2$

(c) $\delta_{\tilde{f}_1(4.71)} = 0.606 \times 10^{-1}, \quad \delta_{\tilde{f}_2(4.71)} = -0.253 \times 10^{-2}$

(d) Algoritmo de Horner (2)

$$u_1 = x + a, \quad u_2 = u_1 \times x, \quad u_3 = u_2 + b, \quad u_4 = u_3 \times x, \quad z = u_5 = u_4 + c$$

$$\delta^L_{\tilde{z}} = \frac{1}{f(x)}\Big[x(3x^2 + 2ax + b)\delta_{\tilde{x}} + ax^2\delta_{\tilde{a}} + bx\delta_{\tilde{b}} + c\delta_{\tilde{c}}$$
$$+x^2(x+a)(\delta_{\text{arr},1} + \delta_{\text{arr},2}) + x(x^2 + ax + b)(\delta_{\text{arr},3} + \delta_{\text{arr},4})\Big] + \delta_{\text{arr},5}$$

[1.17] (a) $x_1 = -69.1055293779093\ldots, \qquad x_2 = -0.014470622090621\ldots$

(b) Alg. 1: $\tilde{x}_1 = -69.10, \qquad \tilde{x}_2 = -0.2000 \times 10^{-1}$

Alg. 2: $\tilde{x}_1 = -69.10, \qquad \tilde{x}_2 = -0.1447 \times 10^{-1}$

(c) Alg. 1: $\delta_{\tilde{x}_1} = 0.800 \times 10^{-4}, \qquad \delta_{\tilde{x}_2} = -0.382$

Alg. 2: $\delta_{\tilde{x}_1} = 0.800 \times 10^{-4}, \qquad \delta_{\tilde{x}_2} = 0.430 \times 10^{-4}$

(d) Alg. 1: $u_1 = b \times b, \qquad u_2 = u_1 - c, \qquad u_3 = \sqrt{u_2}$

$x_1 = u_4 = -b - u_3, \qquad x_2 = u_5 = -b + u_3$

Alg. 2: $u_1 = b \times b, \qquad u_2 = u_1 - c, \qquad u_3 = \sqrt{u_2}$

$x_1 = u_4 = -b - u_3, \qquad x_2 = u_5 = c \div x_1$

Alg. 1: $\delta^L_{\tilde{x}_1} = \dfrac{b}{\Delta}\,\delta_{\tilde{b}} + \dfrac{c}{2x_1\Delta}\,\delta_{\tilde{c}} - \dfrac{b^2}{2x_1\Delta}\,\delta_{\text{arr},1} - \dfrac{\Delta}{2x_1}\,\delta_{\text{arr},2} - \dfrac{\Delta}{x_1}\,\delta_{\text{arr},3} + \delta_{\text{arr},4}$

$\delta^L_{\tilde{x}_2} = -\dfrac{b}{\Delta}\,\delta_{\tilde{b}} - \dfrac{c}{2x_2\Delta}\,\delta_{\tilde{c}} + \dfrac{b^2}{2x_2\Delta}\,\delta_{\text{arr},1} + \dfrac{\Delta}{2x_2}\,\delta_{\text{arr},2} + \dfrac{\Delta}{x_2}\,\delta_{\text{arr},3} + \delta_{\text{arr},5}$

Alg. 2: $\delta^L_{\tilde{x}_1} = \dfrac{b}{\Delta}\,\delta_{\tilde{b}} + \dfrac{c}{2x_1\Delta}\,\delta_{\tilde{c}} - \dfrac{b^2}{2x_1\Delta}\,\delta_{\text{arr},1} - \dfrac{\Delta}{2x_1}\,\delta_{\text{arr},2} - \dfrac{\Delta}{x_1}\,\delta_{\text{arr},3} + \delta_{\text{arr},4}$

$\delta^L_{\tilde{x}_2} = -\dfrac{b}{\Delta}\,\delta_{\tilde{b}} - \dfrac{c}{2x_2\Delta}\,\delta_{\tilde{c}} + \dfrac{b^2}{2x_1\Delta}\,\delta_{\text{arr},1} + \dfrac{\Delta}{2x_1}\,\delta_{\text{arr},2} + \dfrac{\Delta}{x_1}\,\delta_{\text{arr},3} - \delta_{\text{arr},4} + \delta_{\text{arr},6}$

onde $\Delta = \sqrt{b^2 - c}$

$b^2 \gg c: \quad \Delta \approx b, \qquad x_1 \approx -2b, \qquad x_2 \approx -\dfrac{c}{2b}$

Alg. 1: $\delta^L_{\tilde{x}_1} \approx \delta_{\tilde{b}} - \dfrac{c}{4b^2}\,\delta_{\tilde{c}} + \dfrac{1}{4}\left(\delta_{\text{arr},1} + \delta_{\text{arr},2} + 2\delta_{\text{arr},3}\right) + \delta_{\text{arr},4}$

$\delta^L_{\tilde{x}_2} \approx -\delta_{\tilde{b}} + \delta_{\tilde{c}} - \dfrac{b^2}{c}\left(\delta_{\text{arr},1} + \delta_{\text{arr},2} + 2\delta_{\text{arr},3}\right) + \delta_{\text{arr},5}$

Alg. 2: $\delta^L_{\tilde{x}_1} \approx \delta_{\tilde{b}} - \dfrac{c}{4b^2}\,\delta_{\tilde{c}} + \dfrac{1}{4}\left(\delta_{\text{arr},1} + \delta_{\text{arr},2} + 2\delta_{\text{arr},3}\right) + \delta_{\text{arr},4}$

$\delta^L_{\tilde{x}_2} \approx \delta_{\tilde{b}} + \delta_{\tilde{c}} - \dfrac{1}{4}\left(\delta_{\text{arr},1} + \delta_{\text{arr},2} + 2\delta_{\text{arr},3}\right) - \delta_{\text{arr},4} + \delta_{\text{arr},6}$

[1.20] **(b)** Sem p.p.p.: $\tilde{x} = -10.00, \qquad \tilde{y} = 1.001$

Com p.p.p.: $\tilde{x} = 10.00, \qquad \tilde{y} = 1.000$

(c) Sem p.p.p.: $\delta_{\tilde{x}} = 2.00 \qquad \delta_{\tilde{y}} = -0.001$

Com p.p.p.: $\delta_{\tilde{x}} = 0.00, \qquad \delta_{\tilde{y}} = 0.00$

(d) $$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} - \left(\dfrac{a_{21}}{a_{11}}\right) a_{12} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 - \left(\dfrac{a_{21}}{a_{11}}\right) b_1 \end{bmatrix}$$

Algoritmo: $u_1 = \dfrac{a_{21}}{a_{11}}, \qquad u_2 = u_1 \times a_{12}, \qquad u_3 = u_1 \times b_1,$

$u_4 = a_{22} - u_2, \qquad u_5 = b_2 - u_3, \qquad y = u_6 = u_5 \div u_4,$

$u_7 = a_{12} \times y, \qquad u_8 = b_1 - u_7, \qquad x = u_9 = u_8 \div a_{11}$

$$\delta_{\tilde{y}}^L = \frac{u_2}{u_4}(\delta_{\text{arr},1} + \delta_{\text{arr},2}) - \frac{u_3}{u_5}(\delta_{\text{arr},1} + \delta_{\text{arr},3}) - \delta_{\text{arr},4} + \delta_{\text{arr},5} + \delta_{\text{arr},6}$$

$$\delta_{\tilde{x}}^L = -\frac{u_7}{u_8}(\delta_{\tilde{y}}^L + \delta_{\text{arr},7}) + \delta_{\text{arr},8} + \delta_{\text{arr},9}$$

[2.5] **(a)** convergência logarítmica ou infralinear, $K_\infty^{[1]} = 0$;

(b) convergência linear, $K_\infty^{[1]} = 1$;

(c) convergência (supralinear) de ordem b, $K_\infty^{[b]} = 1$;

(d) convergência exponencial, $K_\infty^{[r]} = 0, \ \forall r \geq 1$;

(e) convergência linear, $\dfrac{1}{12} \leq K_n^{[1]} \leq \dfrac{3}{4}$.

[2.7] $x_m = -2 \times 4^m + (3 + 2m) \times 2^m$

[3.3] (b) $z = -3.1831 \pm 0.5 \times 10^{-4}$

(c) $n \geq 19$

[3.5] (c) $z = 0.71481 \pm 0.5 \times 10^{-5}$

[3.6] (b) $z = 4.30658 \pm 0.5 \times 10^{-5}$

[3.8] (b)

Método	Pontos fixos		O.c.	$K_\infty^{[r]}$
	Todos	Método converge	r	
1	$\frac{6}{5},\ 2$	$-$	$-$	$-$
2	$-4,\ 3$	3	1	$\frac{3}{4}$
3	$\sqrt[3]{3}$	$\sqrt[3]{3}$	2	$\frac{1}{\sqrt[3]{3}}$
4	$-\sqrt{a},\ 0,\ \sqrt{a}$	$-\sqrt{a},\ \sqrt{a}$	3	$\frac{1}{4a}$

[3.24] (c) $z_1 = 1.139194147 \pm 0.5 \times 10^{-9}$

[3.25] (b) $z_2 = 2.745898312 \pm 0.5 \times 10^{-9}$

[3.26] (b) $\alpha = 0.851241066782 \pm 0.5 \times 10^{-12}$

[3.27] (b) $q = \dfrac{1-p}{2}$

(d) $\sqrt[3]{231} = 6.135792439661959 \pm 0.5 \times 10^{-15}$

[3.35] (b) $z_3 = 5.114907541477 \pm 0.5 \times 10^{-12}$

[3.36] (b) $z = 1.126561908150 \pm 0.5 \times 10^{-12}$

[4.14]

(a) $A^{-1} = \begin{bmatrix} 62 & -36 & -19 \\ -36 & 21 & -11 \\ -19 & 11 & 6 \end{bmatrix}$

(b) $\sigma(A) = \{0.0112673,\ \ 5.32658,\ \ 16.6622\}$

(c) $\text{cond}_1(A) = \text{cond}_\infty(A) = 2340, \qquad \text{cond}_2(A) = 1478.81$

[4.15] **(b)** $\text{cond}_1(A) = \text{cond}_\infty(A) = (2\alpha+1)^2, \qquad \text{cond}_2(A) = (\alpha+\beta)^2$

(c) $x = u_1, \qquad \tilde{x} = \left(1 + \dfrac{\varepsilon}{\lambda_1}\right) u_1$

(d) $x = u_1, \qquad \tilde{x} = u_1 + \dfrac{\varepsilon}{\lambda_2} u_2$

[4.16]

(a) $A^{-1} = \begin{bmatrix} \beta' & \alpha' & \cdots & \cdots & \alpha' \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 & 0 \\ 0 & \cdots & \cdots & 0 & 1 \end{bmatrix}, \qquad \alpha' = -\dfrac{\alpha}{\beta}, \qquad \beta' = \dfrac{1}{\beta}$

(b)

$$\text{cond}_1(A) = \max\{|\beta|, |\alpha|+1\} \times \max\left\{\frac{1}{|\beta|}, 1 + \frac{|\alpha|}{|\beta|}\right\}$$

$$\text{cond}_\infty(A) = \max\{|\beta| + (n-1)|\alpha|, 1\} \times \max\left\{\frac{1+(n-1)|\alpha|}{|\beta|}, 1\right\}$$

$$\text{cond}_*(A) = \max\left\{|\beta|, \frac{1}{|\beta|}\right\}$$

(c) $\|\delta_{\tilde{x}}\|_\infty < \dfrac{2\mu^2}{1-\mu^2}$

[4.24]

Método	Matriz triangular superior	Matrix triangular inferior
Jacobi	n iteradas	n iteradas
Gauss-Seidel	n iteradas	1 iterada

[4.29]

(a) $A' = \begin{bmatrix} 10 & 1 & 5 \\ 1 & 10 & 5 \\ 1 & -3 & -10 \end{bmatrix}, \qquad b' = \begin{bmatrix} 31 \\ 22 \\ -21 \end{bmatrix}.$

(b) $x \approx x^{(5)} = \frac{1}{10^5}\,[200725 \;\; 100726 \;\; 200119]^T$

$\|x - x^{(5)}\|_\infty \leq \frac{4347}{2 \times 10^5} = 0.0217$

(c) $x \approx x^{(3)} = \frac{1}{8 \times 10^6}\,[16018100 \;\; 8050490 \;\; 15986663]^T$

$\|x - x^{(3)}\|_\infty \leq \frac{87453}{16 \times 10^5} = 0.0547$

[4.36] **(a)** $\alpha \in \left]-1, \frac{5}{2}\right[$, **(b)** $\alpha \in \left]-\frac{5}{2}, 1\right[$

(c) $\alpha \in \left]-1, \frac{5}{2}\right[$, **(d)** $\alpha \in]-1, 1[$

[4.40]

$$r_\sigma(\omega) = \begin{cases} \left(\gamma + \sqrt{\gamma^2 + 1 - \omega}\right)^2, & \omega \notin [\omega_-, \omega_+], \\ \omega - 1, & \omega \in [\omega_-, \omega_+] \end{cases}$$

$$\gamma = \frac{\alpha\omega}{2}, \qquad \omega_\pm = \frac{2}{\alpha^2}\left(1 \pm \sqrt{1 - \alpha^2}\right)$$

O método converge para $\omega \in]0, 2[$.

[4.43] $\omega \in]0, 2\omega_{\text{opt}}[, \qquad \omega_{\text{opt}} = \frac{a}{a^2 + b^2}, \qquad r_\sigma(\omega_{\text{opt}}) = \frac{b}{\sqrt{a^2 + b^2}}$

[5.1] **(a)** $z = g(z), \qquad z \in D = \left[0, \frac{1}{2}\right] \times \left[0, \frac{1}{2}\right], \qquad g(x) = \begin{bmatrix} x_2/\sqrt{5} \\ \frac{1}{4}(\sin x_1 + \cos x_2) \end{bmatrix}$

$\tilde{z} = \tilde{g}(\tilde{z}), \qquad \tilde{z} \in \tilde{D} = \left[-\frac{1}{2}, 0\right] \times \left[0, \frac{1}{2}\right] \qquad \tilde{g}(x) = \begin{bmatrix} -x_2/\sqrt{5} \\ \frac{1}{4}(\sin x_1 + \cos x_2) \end{bmatrix}$

$g(g(\mathbb{R}^2)) \subset D, \qquad \tilde{g}(\tilde{g}(\mathbb{R}^2)) \subset \tilde{D}$

(b) $z \approx x^{(4)} = \begin{bmatrix} 0.123016 \\ 0.270318 \end{bmatrix}, \qquad \tilde{z} \approx x^{(4)} = \begin{bmatrix} -0.100147 \\ 0.219418 \end{bmatrix}$

(c) $\|z - x^{(4)}\|_\infty \leq 0.476 \times 10^{-2}, \qquad \|\tilde{z} - x^{(4)}\|_\infty \leq 0.452 \times 10^{-2}$

[5.5] **(a)** $g(x) = \begin{bmatrix} -\frac{1}{2}\left(x_2 + \varepsilon \cos x_3\right) \\ -\frac{1}{3}\left(x_1 + 3\varepsilon x_1 x_3\right) \\ -\frac{1}{3}\left(x_2 + \varepsilon x_1^2\right) \end{bmatrix}$ **(b)** $z \approx x^{(2)} = \begin{bmatrix} -0.125 \\ +0.0416667 \\ -0.00130208 \end{bmatrix}$

(c) 28 iteradas.

[5.9] **(a)** $z_1 = x^{(1)} = \begin{bmatrix} \frac{8}{3} \\ \frac{7}{3} \\ 1 \end{bmatrix}, \quad \beta \neq \frac{8}{3}$ **(b)** $z_2 \approx x^{(2)} = \begin{bmatrix} -\frac{43}{9} \\ -\frac{46}{9} \\ -\frac{29}{6} \end{bmatrix}$

(c) $z_1 = \begin{bmatrix} \frac{8}{3} \\ \frac{7}{3} \\ 1 \end{bmatrix}, \qquad z_2 = \begin{bmatrix} -1 - \frac{8}{3}\sqrt{2} \\ -\frac{4}{3} - \frac{8}{3}\sqrt{2} \\ -2 - 2\sqrt{2} \end{bmatrix}, \qquad z_3 = \begin{bmatrix} -1 + \frac{8}{3}\sqrt{2} \\ -\frac{4}{3} + \frac{8}{3}\sqrt{2} \\ -2 + 2\sqrt{2} \end{bmatrix}$

[5.12] **(a)** $z \approx x^{(2)} = \begin{bmatrix} 1.33636 \\ 1.75424 \end{bmatrix}$

(b) $\|z - x^{(2)}\|_\infty \leq 0.272 \times 10^{-5}$

(c) $\begin{bmatrix} -3.00162 \\ 0.148108 \end{bmatrix}, \qquad \begin{bmatrix} -0.901266 \\ -2.08659 \end{bmatrix}, \qquad \begin{bmatrix} 1.33636 \\ 1.75424 \end{bmatrix}, \qquad \begin{bmatrix} 2.99837 \\ 0.148431 \end{bmatrix}$

[6.14] **(a)** $$p_3(x) = -\frac{1}{6}\,x(x-1)(x-2) - \frac{1}{2}\,x(x+1)(x+2) + \frac{8}{3}\,x(x^2-1) = x(2x^2+x-2)$$

(b) $$p_3(x) = 1 - (x+1) + (x+1)x + 2(x+1)x(x-1) = x(2x^2+x-2)$$

(d) $$|e_3(x)| \le \frac{25}{24}, \quad \forall x \in [-1, 2]$$

(e) $$p_4(x) = x^4$$

(f) $$f(x) = x^4 + k(x+1)x(x-1)(x-2)(x-3), \qquad 0 < k \le \frac{1}{120}$$

[6.20] **(a)** $$q_n(y) = f^{-1}[y_0] + \sum_{k=0}^{n} f^{-1}[y_0, y_1, \ldots, y_k]W_k(y), \qquad W_k(y) = \prod_{i=0}^{k-1}(y - y_i)$$

(b) $z \approx q_3(0) = 0.567143$

[7.1] **(a)** $p_1^*(x) = \dfrac{1}{5}\,(11 + 23x)$

(b) $p_2^*(x) = \dfrac{1}{5}\,(-9 + 3x + 20x^2)$

(c) $p_3^*(x) = -2x + x^2 + 2x^3$

(d) $d(f, p_1^*) = 2\sqrt{\dfrac{89}{5}} \approx 8.43801, \qquad d(f, p_2^*) = \dfrac{6}{\sqrt{5}} \approx 2.68328, \qquad d(f, p_3^*) = 0$

[7.4] $\phi^*(x) = \dfrac{1}{2}\,(1 + \cos(\pi x))$

[7.10] $\phi^*(x) = \dfrac{1}{a^*x + b^*}, \qquad a^* = 0.429322, \qquad b^* = 0.612601$

[7.14] **(a)** $p_2^* = \dfrac{1}{35}\,(7P_0 + 21P_1 + 20\,P_2), \qquad p_2^*(x) = \dfrac{3}{35}\,(-1 + 7x + 10x^2)$

(b) $p_2^* = \dfrac{1}{8}\,(3T_0 + 6T_1 + 4T_2), \qquad p_2^*(x) = \dfrac{1}{8}\,(-1 + 6x + 8x^2)$

(c) $p_2^* = \dfrac{3}{4}\,(H_0 + H_1 + H_2), \qquad p_2^*(x) = \dfrac{3}{4}\,(-1 + 2x + 4x^2)$

(d) $p_2^* = 6\,(5L_0 - 19L_1 + 27L_2), \qquad p_2^*(x) = 3\,(26 - 70x + 27x^2)$

[8.1]

$f(x) = \exp(-x^2), \qquad I(f) = 0.746824132812$

M	$I^{(M)}$	$\dfrac{1}{6M^2}$	$\dfrac{1}{3}\left\|I^{(M)} - I^{(M/2)}\right\|$	$\left\|E^{(M)}\right\|$	$\dfrac{\left\|E^{(M)}\right\|}{\left\|E^{(M/2)}\right\|}$
1	0.683939720586	0.166667		0.628844×10^{-1}	
2	0.731370251829	0.416667×10^{-1}	0.158102×10^{-1}	0.154539×10^{-1}	0.245751
4	0.742984097800	0.104167×10^{-1}	0.387128×10^{-2}	0.384004×10^{-2}	0.248484
8	0.745865614846	0.260417×10^{-2}	0.960506×10^{-3}	0.958518×10^{-3}	0.249612
16	0.746584596788	0.651042×10^{-3}	0.239661×10^{-3}	0.239536×10^{-3}	0.249902
32	0.746764254652	0.162760×10^{-3}	0.598860×10^{-4}	0.598782×10^{-4}	0.249976
64	0.746809163638	0.406901×10^{-4}	0.149697×10^{-4}	0.149692×10^{-4}	0.249994
128	0.746820390542	0.101725×10^{-4}	0.374230×10^{-5}	0.374227×10^{-5}	0.249998
256	0.746823197246	0.254313×10^{-5}	0.935568×10^{-6}	0.935566×10^{-6}	0.250000

$f(x) = \sqrt{x}, \qquad I(f) = \dfrac{2}{3}$

M	$I^{(M)}$	$\dfrac{1}{6M^2}$	$\dfrac{1}{3}\left\|I^{(M)} - I^{(M/2)}\right\|$	$\left\|E^{(M)}\right\|$	$\dfrac{\left\|E^{(M)}\right\|}{\left\|E^{(M/2)}\right\|}$
1	0.500000000000			0.166667	
2	0.603553390593		0.345178×10^{-1}	0.631133×10^{-1}	0.378680
4	0.643283046243		0.132432×10^{-1}	0.233836×10^{-1}	0.370502
8	0.658130221624		0.494906×10^{-2}	0.853645×10^{-2}	0.365061
16	0.663581196877		0.181699×10^{-2}	0.308547×10^{-2}	0.361447
32	0.665558936279		0.659246×10^{-3}	0.110773×10^{-2}	0.359015
64	0.666270811379		0.237292×10^{-3}	0.395855×10^{-3}	0.357357
128	0.666525657297		0.849486×10^{-4}	0.141009×10^{-3}	0.356214
256	0.666616548977		0.302972×10^{-4}	0.501177×10^{-4}	0.355421

[8.8]

$f(x) = \exp(-x^2), \qquad I(f) = 0.746824132812$

n	I_n	$\|I - I_n\|$	$\left\|1 - \dfrac{I_n}{I}\right\|$
1	0.745119412436	0.170×10^{-2}	0.228×10^{-2}
2	0.746830391489	0.626×10^{-5}	0.838×10^{-5}
3	0.746838057512	0.139×10^{-4}	0.186×10^{-4}
6	0.746823756571	0.376×10^{-6}	0.504×10^{-6}

$f(x) = \dfrac{1}{1+x^2}, \qquad I(f) = 0.785398163397$

n	I_n	$\lvert I - I_n \rvert$	$\left\lvert 1 - \frac{I_n}{I} \right\rvert$
1	0.784240766618	0.116×10^{-2}	0.147×10^{-2}
2	0.785397945234	0.218×10^{-6}	0.278×10^{-7}
3	0.785395862445	0.230×10^{-5}	0.293×10^{-5}
6	0.785392713917	0.545×10^{-5}	0.694×10^{-5}

[8.14] **(a)** $A_0 = e - 2, \quad A_1 = 1$

(b) $E_1(f) = -\dfrac{3-e}{2} f''(\xi), \qquad \xi \in [0, 1]$

(c) $I_1(f) = 0.841471, \qquad |E_1(f)| \leq 0.118$

(d) $I_1^{(4)}(f) = 0.923705, \qquad \left| E_1^{(4)}(f) \right| \leq \dfrac{\sqrt{2} e^{\pi/4}}{192} = 0.0162$

(e) $M = 51$

[8.22] $I_0(f) = f(1)$

$$I_1(f) = \frac{2+\sqrt{2}}{4} f(2-\sqrt{2}) + \frac{2-\sqrt{2}}{4} f(2+\sqrt{2})$$

$$I_2(f) = w_0 f(x_0) + w_1 f(x_1) + w_2 f(x_2)$$

$$x_0 = 0.415774556783479 \qquad w_0 = 0.711093009929173$$

$$x_1 = 2.294280360279041 \qquad w_1 = 0.278517733569241$$

$$x_2 = 6.289945082937480 \qquad w_2 = 0.010389256501586$$

[10.9] **(a)** $Y(x_0+h) \approx y_2$, onde

$$y_1 = y_0 + \frac{h}{4}\left[f(x_0,y_0) + f\left(x_0+\frac{h}{2}, y_0+\frac{h}{2}f(x_0,y_0)\right)\right]$$

$$y_2 = y_1 + \frac{h}{4}\left[f(x_1,y_1) + f\left(x_1+\frac{h}{2}, y_1+\frac{h}{2}f(x_1,y_1)\right)\right], \qquad x_1 = x_0+\frac{h}{2}$$

(b) $Y(x_0+h) \approx y_1$, onde

$$y_1 = y_0 + hf(x_0,y_0) + \frac{h^2}{2}\,(d_f f)(x_0,y_0) + \frac{h^3}{6}\,(d_f^2 f)(x_0,y_0) + \frac{h^4}{24}\,(d_f^3 f)(x_0,y_0)$$

$$d_f f = f_x + f f_y$$

$$d_f^2 f = f_{xx} + f_x f_y + 2 f f_{xy} + f f_y^2 + f^2 f_{yy}$$

$$\begin{aligned} d_f^3 f \; &= f_{xxx} + f_{xx} f_y + 3 f_x f_{xy} + 3 f f_{xxy} + f_x f_y^2 + 3 f f_x f_{yy} \\ &\quad + 5 f f_y f_{xy} + 3 f^2 f_{xyy} + f f_y^3 + 4 f^2 f_y f_{yy} + f^3 f_{yyy} \end{aligned}$$

(c) $Y(x_0+h) \approx y_1$, onde

$$y_1 = y_0 + \frac{h}{6}\,[\varphi_1 + 2\varphi_2 + 2\varphi_3 + \varphi_4]$$

$$\varphi_1 = f(x_0,y_0), \qquad \varphi_2 = f\left(x_0+\frac{h}{2}, y_0+\frac{h}{2}\,\varphi_1\right)$$

$$\varphi_3 = f\left(x_0+\frac{h}{2}, y_0+\frac{h}{2}\,\varphi_2\right), \qquad \varphi_4 = f(x_0+h, y_0+h\varphi_3)$$

[10.17]

$$W(x) = \begin{bmatrix} y(x) \\ y'(x) \\ y''(x) \end{bmatrix} = \begin{bmatrix} y(x) \\ z(x) \\ w(x) \end{bmatrix}$$

$$W'(x) = \begin{bmatrix} z(x) \\ w(x) \\ f(x, y(x), z(x), w(x)) \end{bmatrix} = F(x, W(x)), \qquad W(x_0) = \begin{bmatrix} y_0 \\ z_0 \\ w_0 \end{bmatrix} = W_0$$

(a) $Y(x_0+h) \approx y_2, \qquad Y'(x_0+h) \approx z_2, \qquad Y''(x_0+h) \approx w_2$

$$W_1 = \begin{bmatrix} y_0 + \frac{h}{2}\, z_0 \\ z_0 + \frac{h}{2}\, w_0 \\ w_0 + \frac{h}{2}\, f(x_0, y_0, z_0, w_0) \end{bmatrix} = \begin{bmatrix} y_1 \\ z_1 \\ w_1 \end{bmatrix}$$

$$W_2 = \begin{bmatrix} y_1 + \frac{h}{2}\, z_1 \\ z_1 + \frac{h}{2}\, w_1 \\ w_1 + \frac{h}{2}\, f(x_1, y_1, z_1, w_1) \end{bmatrix} = \begin{bmatrix} y_2 \\ z_2 \\ w_2 \end{bmatrix}, \qquad x_1 = x_0 + \frac{h}{2}$$

(b) $Y(x_0 + h) \approx y_1, \qquad Y'(x_0 + h) \approx z_1, \qquad Y''(x_0 + h) \approx w_1$

$$W_1 = \begin{bmatrix} y_0 + hz_0 + \frac{h^2}{2}\, w_0 \\ z_0 + hw_0 + \frac{h^2}{2}\, f(x_0, y_0, z_0, w_0) \\ w_0 + hf(x_0, y_0, z_0, w_0) + \frac{h^2}{2}\,(f_x + zf_y + wf_z + ff_w)(x_0, y_0, z_0, w_0) \end{bmatrix} = \begin{bmatrix} y_1 \\ z_1 \\ w_1 \end{bmatrix}$$

(c) $Y(x_0 + h) \approx y_1, \qquad Y'(x_0 + h) \approx z_1, \qquad Y''(x_0 + h) \approx w_1$

$$\tilde{x}_1 = x_0 + \frac{2h}{3}, \qquad \tilde{W}_1 = \begin{bmatrix} y_0 + \frac{2h}{3}\, z_0 \\ z_0 + \frac{2h}{3}\, w_0 \\ w_0 + \frac{2h}{3}\, f(x_0, y_0, z_0, w_0) \end{bmatrix} = \begin{bmatrix} \tilde{y}_1 \\ \tilde{z}_1 \\ \tilde{w}_1 \end{bmatrix}$$

$$W_1 = \begin{bmatrix} y_0 + \frac{h}{4}\,[z_0 + 3\tilde{z}_1] \\ z_0 + \frac{h}{4}\,[w_0 + 3\tilde{w}_1] \\ w_0 + \frac{h}{4}\,[f(x_0, y_0, z_0, w_0) + 3f(\tilde{x}_1, \tilde{y}_1, \tilde{z}_1, \tilde{w}_1)] \end{bmatrix}$$

(d) $Y(x_0 + 2h) \approx y_2, \quad Y'(x_0 + 2h) \approx z_2, \quad Y''(x_0 + 2h) \approx w_2$

$$W_2^{(0)} = \begin{bmatrix} y_1 + \frac{h}{2}\,[3z_1 - z_0] \\ z_1 + \frac{h}{2}\,[3w_1 - w_0] \\ w_1 + \frac{h}{2}\,[3f(x_1, y_1, z_1, w_1) - f(x_0, y_0, z_0, w_0)] \end{bmatrix} = \begin{bmatrix} y_2^{(0)} \\ z_2^{(0)} \\ w_2^{(0)} \end{bmatrix}$$

$$W_2 = \begin{bmatrix} y_1 + \frac{h}{2}\left[z_2^{(0)} + z_1\right] \\ z_1 + \frac{h}{2}\left[w_2^{(0)} + w_1\right] \\ w_1 + \frac{h}{2}\left[f(x_2, y_2^{(0)}, z_2^{(0)}, w_2^{(0)}) + f(x_1, y_1, z_1, w_1)\right] \end{bmatrix} = \begin{bmatrix} y_2 \\ z_2 \\ w_2 \end{bmatrix}$$

[10.25] $y_{n+1} = a_0 y_n + a_1 y_{n-1} + a_2 y_{n-2} + h[b_{-1} f_{n+1} + b_0 f_n + b_1 f_{n-1} + b_2 f_{n-2}]$

(i) Condições para que o método tenha ordem de consistência ≥ 3:

$C_0 = C_1 = C_2 = C_3 = 0$

$$\begin{cases} a_2 = 1 - a_0 - a_1 \\ b_0 = \dfrac{1}{12}(27 - 4a_0 + a_1 - 36b_{-1}) \\ b_1 = \dfrac{1}{3}(-4a_0 - 2a_1 + 9b_{-1}) \\ b_2 = \dfrac{1}{12}(9 - 4a_0 - 5a_1 - 12b_{-1}) \end{cases}$$

$$\begin{cases} C_4 = 9 - a_1 - 24b_{-1}) \\ C_5 = \dfrac{1}{3}(-81 + 4a_0 + 17a_1 + 180b_{-1}) \end{cases}$$

(ii) Condição da raiz:

$$\rho(r) = r^3 - a_0r^2 - a_1r - a_2 = (r-1)[r^2 + (1-a_0)r + (1 - a_0 - a_1)]$$

Triângulo de convergência:

$$T = \{(a_0, a_1) \in \mathbb{R}^2 : a_1 > -a_0 \wedge a_1 \leq 1 \wedge a_1 < 3 - 2a_0\}$$

(iii) Condições para que o método tenha ordem de consistência ≥ 4:

$$C_4 = 0$$

$$\begin{cases} b_{-1} = \dfrac{9 - a_1}{24} \\ a_2 = 1 - a_0 - a_1 \\ b_0 = \dfrac{1}{24}(27 - 8a_0 + 5a_1) \\ b_1 = \dfrac{1}{24}(27 - 32a_0 - 19a_1) \\ b_2 = \dfrac{1}{24}(9 - 8a_0 - 9a_1) \end{cases}$$

$$C_5 = \frac{1}{6}(-27 + 8a_0 + 19a_1)$$

(iv) Condições para que o método tenha ordem de consistência ≥ 5:

$$C_5 = 0 \quad \Leftrightarrow \quad 8a_0 + 19a_1 = 27$$

Não há nenhum método convergente de ordem ≥ 5 pois esta recta não intersecta o triângulo de convergência T.