Algebraic Geometry Course

Section 0

Introduction

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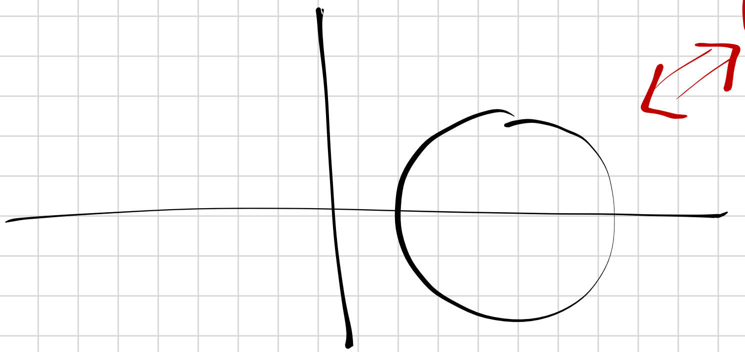
- **What?** Geometry described through algebra, algebra interpreted through geometry

Fermat & Descartes XVII → Shapes can be encoded in equations



$(x-3)^2+y^2=4$

- **Why?** Algebra will provide tools to study these objects better when
  - dealing with singularities
  - higher dimensions
  - looking for numerical invariants
  - studying classifying problems

To maximize the power of algebra: Algebraically closed fields

The ring of functions

One of the most important features of a geometrical object is the functions that can be considered on it

We classify the different branches of geometry by the class of functions

- $C^\infty \Longrightarrow$ differential geometry
- complex analytic functions $\Longrightarrow$ complex analytic geometry
- algebraic functions (polynomials) $\Longrightarrow$ algebraic geometry

$\Updownarrow$

$K[x_1 \ldots x_n]$

- How much do loose by restricting ourselves to algebraic functions?

No that much

Riemann th $\Rightarrow$ compact Riemann surfaces are algebraic

Chow's th $\Rightarrow$ closed subsets (usual top) in $\mathbb{P}^n$ are algebraic

GAGA $\Longrightarrow$ eq. of cats. analytic and algebraic sheaves

A guiding problem

Classification problem: - Describe a class of geometrical objects (varieties...) up to equivalence relations (usually $\cong$, birational eq.)

- Can we construct a space whose pts are equivalence classes?
- Does this space inherit a geometrical structure?

Every classification problem has two parts

- A discrete part $\left\{ \begin{array}{l} \text{numerical invariants} \\ \Uparrow \\ \text{COHOMOLOGY} \end{array} \right.$
- A continuous part (group actions, quotients...)