POSSIBLE PROJECTS FOR THE ALGEBRAIC GEOMETRY COURSE (GALG132646)

Due date: 22nd June 2021.

Description

The students should write a short dissertation about one of the topics listed below, between 10 and 20 pages. It is expected that the student shows a good understanding of the subject and its context, as well as a good handling of the tools and techniques used in the field.

Possible topics

Blow-up and the normal cone degeneration: The projective analog of the Spec construction is the Proj of a graded ring. With this tool at hand, one can revisit the blow-up construction in terms of the Proj of a particular algebra naturally built out of the ideal sheaf of the subvariety that we want to blow-up. Having a good understanding of these techniques, one can easily produce a degeneration of a variety into the cone given by the normal bundle of the subvariety.

Chow's Theorem: states that a closed analytic subvariety of $\mathbb{P}^n$ is always algebraic, constituting a preliminary version of Serre's GAGA Theorem. This theory shows that one can use the power of algebraic geometry to describe analytic geometry, which is the framework of most of physical theories. The recommended reference is Chapter III of [3].

Deformation theory of coherent sheaves: provides a description of the infinitesimal changes that one perform to a particular coherent sheaf. The main tool is the study of families of sheaves parametrized by fat points. This theory is of great importance in moduli theory, as it provides a description of their tangent spaces. The recommended reference is [4].

Intersection theory of curves in a surface: provides a handful of tools to study numerical properties of surfaces and the curves that they contain, such as Riemann-Roch theorem or the genus formula. The recommended reference is Chapter I of [2].

Zariski's Main Theorem: is an important tool to state that two varieties are isomorphic (and not only birrationally equivalent). It requires the introduction of formal functions. The recommended reference is Chapter III.11 of [1].

Linear systems and Bertini's Theorem: states (roughly) that, if a family of curves includes a smooth curve, then the general member of the family is a smooth curve as well. The recommended reference is Chapter II.8 of [1].

The derived category of coherent sheaves: encodes a lot of information about the variety, although it is less rigid than the category of coherent sheaves itself, and a more natural framework, although it requires a great amount of abstraction. The student is expected to construct the derived category, and to show that occurrence of Grothendieck-Verdier dualities in some cases.

Weil cohomology theories: in a previous step towards the understanding of motives, one should study the natural properties that carry the cohomology theory with coefficients on a field. These properties are the Künneth formula, Poincaré duality and the Hard Lefschetz theorem, which should be studied by the student in the case of smooth varieties.

Integral functors: constitute a very important class of functors between the derived categories of schemes, and often, the unique source of derived equivalences. The student should have some knowledge about the derived category, or be able to cover this during the project.

References

[1] R. Hartshorne, *Algebraic geometry.*
[2] A. Beauville, *Complex algebraic surfaces.*
[3] P. Griffiths and J. F. Adams, *Topics in algebraic and analytic geometry.*
[4] N. Nitsure, *Deformation theory for vector bundles.*