

# A logical view of the Banach-Steinhaus and the open mapping theorems I

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## Abstract

We generalize the bounded functional interpretation of Peano arithmetic to base types  $0$  and  $X$ , where  $X$  stands for a normed vector space. The characteristic principles supported by this interpretation inject some uniformities into Peano Arithmetic, e.g. they ensure the principle of collection for bounded matrices. In general, collection for universal matrices does not have a bounded interpretation. Nevertheless, we show that the Banach-Steinhaus and the open mapping theorems of functional analysis can be seen as instances of collection for universal matrices that do have a bounded interpretation. In the process we present a logical version of the Baire category theorem.