

Respostas

1.1: a) $\frac{x^6}{6}$; b) $\frac{x^2}{2} + \frac{2}{3}x^{\frac{3}{2}}$; c) $6\sqrt{x} + \frac{1}{10}x^{\frac{5}{2}}$; d) $-\frac{1}{3}x^{-3} - \frac{1}{2}x^{-2} - \frac{4}{5}x^{-\frac{5}{2}}$; e) $\frac{x^7}{7} + \frac{3}{5}x^5 + x^3 + x$; f) $\frac{2^x}{\log 2}$.

1.2: a) $-\frac{5}{8}(1 - 2x)^{\frac{4}{5}}$; b) $\arcsen \frac{x}{2}$; c) $\operatorname{tg} x$; d) $\frac{\sqrt{5}}{5} \operatorname{arctg} x\sqrt{5}$; e) e^{x+3} ; f) $\operatorname{tg} x - x$.

1.3: a) $-\frac{1}{2} \cos 2x$; b) $\frac{1}{5}e^{5x}$; c) $-\frac{1}{2} \cos x^2$; d) $\frac{1}{2} \log(1 + x^2)$; e) $-\log |\cos x|$; f) $\log |\operatorname{sen} x|$; g) $-\frac{1}{3} \operatorname{cotg} 3x$.

2.1: a) $\frac{1}{3} \log |3x - 7|$; b) $-\frac{1}{2} \log |\cos 2x|$; c) $\frac{1}{5} \log |\operatorname{sen}(5x - 7)|$; d) $\frac{1}{2} \operatorname{tg}^2 x$; e) $-\frac{1}{4} \cos^4 x$; f) $-\frac{1}{\operatorname{sen} x}$; g) $\log(2 + e^x)$.

2.2: a) $\frac{1}{2} \log(1 + x^2)$; b) $\frac{1}{4} \operatorname{arctg} x^4$; c) $\frac{1}{4} \operatorname{sh}^2(2x + 1)$ ou $\frac{1}{4} \operatorname{ch}^2(2x + 1)$; d) $\frac{3^{\operatorname{sen}^2 x}}{\log 3}$; e) $-2 \log |\cos \sqrt{x}|$; f) $\arcsen e^x$; g) $\frac{\operatorname{tg}^2 x}{2} + \log |\cos x|$. h) $\operatorname{arccosh} x$.

2.3: a) $\log |x-5|$; b) $-\frac{3}{x+2}$; c) $\frac{1}{2} \operatorname{arctg} \frac{x}{2}$; d) $\frac{1}{2} \log(x^2+4)$; e) $\frac{2}{\sqrt{3}} \operatorname{arctg} \left(\frac{2}{\sqrt{3}} \left(x + \frac{1}{2} \right) \right)$; f) $\frac{1}{2} \log |x^2 + x + 1| - \frac{1}{\sqrt{3}} \operatorname{arctg} \left(\frac{2}{\sqrt{3}} \left(x + \frac{1}{2} \right) \right)$; g) $\frac{1}{2} \log \frac{|x-1|}{|x+1|}$; h) $2 \log |x - 1| - \log |x + 1| - \log |x|$; i) $-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \log |x - 1|$.

2.4: a) $\frac{x}{2} - \frac{1}{4} \operatorname{sen} 2x$; b) $\frac{x}{2} + \frac{1}{4} \operatorname{sen} 2x$; c) $-x - \operatorname{cotg} x$; d) $\log |\sec x + \operatorname{tg} x|$; e) $\log |\operatorname{cosec} x - \operatorname{cotg} x|$; f) $-\cos x + \frac{\cos^3 x}{3}$; g) $\frac{\operatorname{sen}^3 x}{3} - \frac{\operatorname{sen}^5 x}{5}$.

2.5: a) $e^x(x - 1)$; b) $x(\log x - 1)$; c) $\frac{e^x}{2}(\operatorname{sen} x + \cos x)$; d) $-x^2 \cos x + 2x \operatorname{sen} x + 2 \cos x$; e) $x \operatorname{arctg} x - \frac{1}{2} \log(1 + x^2)$; f) $\frac{x}{2} (\cos(\log x) + \operatorname{sen}(\log x))$.

2.6: $\frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2} \log |x + \sqrt{1+x^2}|$.

3.1: a) $\frac{1}{4}$; b) $e^e - e$; c) π ; d) 0 .

3.2: a) $-\log 2 + \log |1 - \sqrt{2}| - \log |1 + \sqrt{2}|$; b) $\frac{5\sqrt{15}-\sqrt{3}}{4}$; c) $\frac{\pi}{6}$; d) $-\frac{e^4}{2} - \frac{5}{2}e^2 + 3e + 3 \log \frac{e^2-1}{e-1}$.

3.3: i) $S_d = \frac{1}{2} = \int_0^1 f(x)dx$; ii) $S_d = \frac{4}{3} (\int_0^2 f(x)dx = 0)$; iii) $S_d = \frac{28}{25} = \frac{140}{125}$
 $(\int_0^1 f(x) dx = \frac{144}{125})$; iv) $S_d = \frac{\sqrt{3}}{8}\pi (\int_0^\pi f = 0)$.

3.4: Com $c_k = \frac{2k-1}{2n}$ vem $S_{d_n} = -\frac{3}{2} = \int_0^1 -3xdx$. Com $c_k = x_k$ vem $S_{d_n} = -\frac{3}{2} \frac{n+1}{n}$ (tem o mesmo limite) .

3.5: a) $\frac{1}{4} \log \left| \frac{x}{x-2} \right| - \frac{3}{2(x-2)}$; b) $\frac{1}{2} \log |x+1| - \frac{1}{4} \log(x^2+1) + \frac{1}{2} \operatorname{arctg} x$; c)
 $-\frac{1}{8x^2} - \frac{1}{4x} - \frac{1}{8} \operatorname{arctg} \frac{x}{2} - \frac{\log|x|}{16} + \frac{1}{32} \log(4+x^2)$; d) $-\frac{1}{3(x-1)} - \frac{\log|x-1|}{3} + \frac{1}{6} \log|x^2+$
 $x+1| + \frac{1}{3\sqrt{3}} \operatorname{arctg} \left(\frac{2}{\sqrt{3}} \left(x + \frac{1}{2} \right) \right)$; e) $\frac{1}{2} \operatorname{arctg} x + \frac{x}{2(x^2+1)}$.

3.6: $\log \left| \operatorname{tg} \frac{x}{2} + 1 \right|$

4.1: $-\frac{e^2}{4} + \frac{3e^4}{4}$.

4.2: a) $F'(x) = -\operatorname{sen}^2 x$; b) $F'(x) = 2x \log(x^4+1) - \log(x^2+1)$; c)
 $\frac{e^{2x}}{x^2+1} + e^x \int_0^x \frac{e^t}{t^2+1} dt$.

4.4: $f(x) = \log(e^x+1) - x(\log(2+2e)-1) - \log 2$.

4.5: $\frac{1}{4}$.

4.6: $-1 + \frac{\pi^2}{8}$

5.1: $4 \left(-4 - 5 \operatorname{arcsen} \frac{1}{\sqrt{10}} + 5 \operatorname{arcsen} \frac{3}{\sqrt{10}} \right)$

5.2: Área: $\frac{9}{2}$; comprimento do bordo: $3\sqrt{2} + \frac{1}{4} (2\sqrt{5} + \operatorname{sh}^{-1} 2 + 4\sqrt{17} + \operatorname{sh}^{-1} 4)$

5.3: Área: $\frac{3-e}{2}$; comprimento do bordo: $\sqrt{e^2-2e+2} + 1 + \sqrt{1+e^2} - \sqrt{2} +$
 $\log(1+\sqrt{2}) - \log(1+\sqrt{1+e^2})$

5.4: 12

5.5: $\frac{2}{3} + \sqrt{3}$

5.6: $\frac{4}{3}\pi ab^2$

5.7: a) 8π ; b) 9π c) $\frac{3\pi}{10}$.