Winter School 2016

Physics and Computation

— Session 2: Physics and Pataphysics —

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Sensorium Dei

Newton-Bentley's correspondence

Letter 1

Newton-Bentley's correspondence led Newton to abandon the Stoic Cosmos of a finite distribution of matter in infinite space and to adopt the Atomist Universe in which matter is distributed throughout infinite space.

If the distribution of matter were finite, then the matter on the outside of this space would by its gravity tend toward the matter on the inside, and by consequence, fall down into the middle of the whole space, and there compose one great spherical mass... But if the matter was evenly diffused through an infinite space, it would never convene into one mass but some of it into one mass and some of it into another so as to make an infinite number of great masses scattered at great distances from one to another throughout all of infinite space. And thus might the Sun and fixed stars be formed.

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The hypothesis of deriving the frame of the world by mechanical principles from matter evenly spread through the heavens being inconsistent with my system, I had considered it very little before your letters put me upon it, and therefore trouble you with a line or two more about it...

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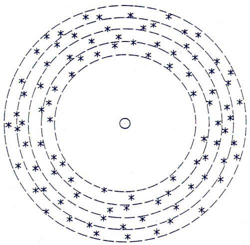

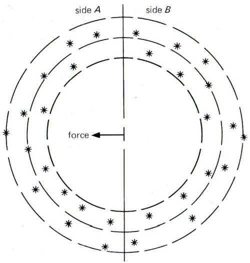

The problem of stability

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Summing Up

Hilbert's tenth problem

Hilbert's tenth problem is the tenth on the list of Hilbert's problems of 1900. Its statement is as follows:

Given a Diophantine equation with any number of unknown quantities and with rational integral numerical coefficients: To devise a process according to which it can be determined in a finite number of operations whether the equation is solvable in rational integers.

Diophantine sets

Definition (Diophantine sets)

We say that a relation D is Diophantine if there exists a polynomial p with integer coefficients, such that,

$$m \in D \text{ iff } \exists x_1, \ldots, x_n \in \mathbb{N}_1 \; [\; p(m, x_1, \ldots, x_n) = 0 \;]$$

Example (Composite numbers, divisible numbers, prime numbers, etc.)

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Example (Composite numbers, divisible numbers, prime numbers, etc.)

$$x \in Composite \quad \text{iff} \quad \exists y, z \in \mathbb{N}_1 \ [\ (y+1)(z+1) - x = 0\ ]$$

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Example (Composite numbers, divisible numbers, prime numbers, etc.)

$$x | y \quad \text{iff} \quad \exists z \in \mathbb{N}_1 \; [\; xz - y = 0 \;]$$

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Example (Composite numbers, divisible numbers, prime numbers, etc.)

$$x|y \textit{ and } x < y \quad \text{iff} \quad \exists u, v \in \mathbb{N}_1 \; [\; (xu - y)^2 + (y - x - v)^2 = 0 \;]$$

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Example (Composite numbers, divisible numbers, prime numbers, etc.)

$$x \textit{ is not a power of } 2 \quad \text{iff} \quad \exists y, z \in \mathbb{N}_1 \; [\; x - y(2z+1) = 0 \;]$$

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Example (Composite numbers, divisible numbers, prime numbers, etc.)

$k + 2 \; is \; prime$ iff $\ldots$

Primes

$$\exists a\ \exists b\ \exists c\ \exists d\ \exists e\ \exists f\ \exists g\ \exists h\ \exists i\ \exists j\ \exists \ell\ \exists m\ \exists n\ \exists o\ \exists p\ \exists q\ \exists r\ \exists s\ \exists t\ \exists u\ \exists v\ \exists x\ \exists w\ \exists y\ \exists z$$

$$
\begin{aligned}
&[wz + h + j - q]^2 \\
&\quad + \quad [(g\mathbf{k} + 2g + \mathbf{k} + 1)(h + j) + h - z]^2 \\
&\quad + \quad [16(\mathbf{k} + 1)^3(\mathbf{k} + 2)(n + 1)^2 + 1 - f^2]^2 \\
&\quad + \quad [2n + p + q + z - e]^2 \\
&\quad + \quad [e^3(e + 2)(a + 1)^2 + 1 - o^2]^2 \\
&\quad + \quad [(a^2 - 1)y^2 + 1 - x^2]^2 \\
&\quad + \quad [16r^2y^4(a^2 - 1) + 1 - u^2]^2 \\
&\quad + \quad [n + \ell + v - y]^2 \\
&\quad + \quad [(a^2 - 1)\ell^2 + 1 - m^2]^2 \\
&\quad + \quad [ai + \mathbf{k} + 1 - \ell - i]^2 \\
&\quad + \quad [((a + u^2(u^2 - a))^2 - 1)(n + 4dy)^2 + 1 - (x + cu)^2]^2 \\
&\quad + \quad [p + \ell(a - n - 1) + b(2an + 2a - n^2 - 2n - 2) - m]^2 \\
&\quad + \quad [q + y(a - p - 1) + s(2ap + 2a - p^2 - 2p - 2) - x]^2 \\
&\quad + \quad [z + p\ell(a - p) + t(2ap - p^2 - 1) - pm]^2) \\
&\quad = \quad 0
\end{aligned}
$$

4 degrees are enough ([V.M93])

$$4x^3y + 5z = 2x^2z^3 + 3y^2x$$

$$p_1 = 4x \qquad p_2 = p_1x \qquad p_3 = p_2x \qquad p_4 = p_3y$$

$$q_1 = 5z$$

$$r_1 = 2x \qquad r_2 = r_1x \qquad r_3 = r_2z \qquad r_4 = r_3z \qquad r_5 = r_4z$$

$$s_1 = 3y \qquad s_2 = s_1y \qquad s_3 = s_2x$$

$$t_1 = p_4 + q_1 \qquad u_1 = r_5 + s_3 \qquad t_1 = u_1$$

Theorem

To solve Hilbert's Tenth Problem positively, it is sufficient to find a method for deciding whether a Diophantine equation of degree 4 has a solution.

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RDP conjecture

Conjetura RDP

A função exponencial (de expressão x^y) é diofantina.

Teorema (Yuri Matiyasevich, January 1970 ([V.M93]))

O conjunto dos números de Fibonacci é diofantino,
ou (alternativamente)...
$m = n^k$ *se e só se as equações numeradas de I a XIII têm solução nas demais variáveis.*

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RDP conjecture

I $x^2 - (a^2 - 1)y^2 = 1$

II $u^2 - (a^2 - 1)v^2 = 1$

III $s^2 - (b^2 - 1)t^2 = 1$

IV $v = ry^2$

V $b = 1 + 4py = a + qu$

VI $s = x + cu$

VII $t = \mathbf{k} + 4(d - 1)y$

VIII $y = \mathbf{k} + \ell - 1$

IX $a = z + 1$

X $(x - y(a - \mathbf{n}) - \mathbf{m})^2 = (f - 1)^2(2a\mathbf{n} - \mathbf{n}^2 - 1)^2$

XI $\mathbf{m} + g = 2a\mathbf{n} - \mathbf{n}^2 - 1$

XII $w = \mathbf{n} + h = \mathbf{k} + \ell$

XIII $a^2 - (w^2 - 1)(w - 1)^2 z^2 = 1$

MRDP theorem

Theorem (Matiyasevich, Robinson, Davis, Putnam ([Dav73, V.M93]))

A set is Diophantine if and only if it is semidecidable.

Undecidability in Physics I

Some problems in Analysis

Assumptions

1. *Let $\mathcal{E}$ be a set of expressions denoting real, single valued, partially defined functions of one variable and let Φ be the set of functions denoted by expressions in $\mathcal{E}$.*
2. *Φ contains the identity function, the rational numbers, and is closed under addition, subtraction, multiplication, and composition.*

The two big problems

1. The identity problem for $(\mathcal{E}, \Phi)$ is the problem of deciding, given $A \in \mathcal{E}$, whether $A(y) \equiv 0$.
2. The integration problem for $(\mathcal{E}, \Phi)$ is the problem of deciding, given $A \in \mathcal{E}$, whether there is function $f \in \Phi$ such that $f'(y) \equiv A(y)$.

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Elementary and subelementary

Conditions on Φ

1. Φ *contains* π *and the real-valued functions* $\sin(y)$ *and* e^y*;*
2. Φ *contains* μ *such that* $\mu(y) = |y|$ *for* $y \neq 0$*;*
3. Φ *contains* β*, a totally defined function such that no* $f \in \Phi$ *and no interval* $\mathcal{I}$ *are such that* $f' \equiv \beta$ *in* $\mathcal{I}$*.*

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Undecidability in Analysis

Theorem

1. *If* Φ *satisfies conditions* 1 *and* 2*, then the identity problem for* $(\mathcal{E}, \Phi)$ *is undecidable;*
2. *If* Φ *satisfies conditions* 1*,* 2 *and* 3*, then the integration problem for* $(\mathcal{E}, \Phi)$ *is undecidable.*

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Undecidability in Analysis

In[1]:= `Simplify[2 Tan[x] / (1 + Tan[x] ^ 2)]`

Out[1]= `Sin[2 x]`

In[2]:= `Simplify[(x - 1) (x + 1) (x ^ 2 + 1) + 1]`

Out[2]= x^4

In[4]:= $\text{Simplify}\left[\frac{1}{3\,(1+x)} - \frac{-1+2\,x}{6\,\left(1-x+x^2\right)} + \frac{2}{3\,\left(1+\frac{1}{3}\,(-1+2\,x)^2\right)}\right]$

Out[4]= $\frac{1}{1+x^3}$

Some problems in Analysis

Just show that

$$\sum_{i=0}^{n}\sum_{j=0}^{i} x_i x_j = \frac{1}{2}\left((\sum_{i=0}^{n} x_i)^2 + \sum_{i=0}^{n} x_i^2 \right) .$$

Just compute

$$\int \frac{(x+1)\log(x)}{(x^2+2x+10)^3} dx$$

Some problems in Analysis

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Just compute

$$\int \frac{(x+1)\log(x)}{(x^2+2x+10)^3} dx$$

Existence

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is the smallest class of expressions obtained by iteration of addition, subtraction, multiplication, and composition, starting with y, e^y, $\sin(y)$ and $|y|$, and expressions for the rational numbers.

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is the class of functions of a real variable usually denoted by the expressions above; take $\beta(y) = e^{y^2}$ and $\mu(y) = |y|$.

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The analytic machinery

$$f[p](m, y_1, \ldots, y_n) = (n+1)^4 \{p(m, y_1, \ldots, y_n)^2 + \sum_{i=1}^{n} \sin^2(\pi y_i) \; (\; g_i(m, y_1, \ldots, y_n) \;)^4\}$$

$$F[p](m, y_1, \ldots, y_n) = f[p](m, y_1^2, \ldots, y_n^2)$$

The analytic machinery

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F[p](m, y_1, \ldots, y_n) &= f[p](m, y_1^2, \ldots, y_n^2)
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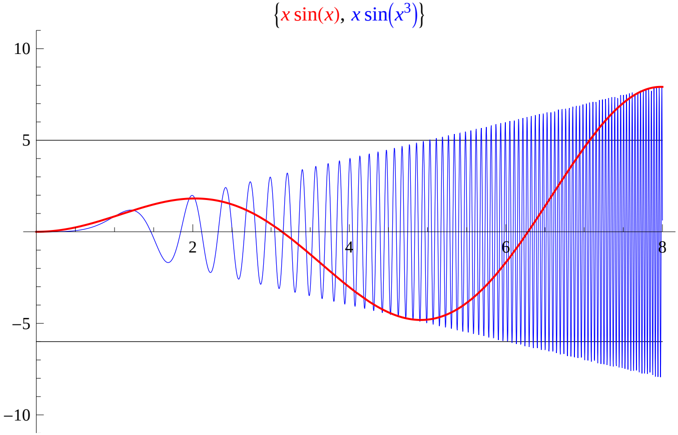
The analytic machinery

Theorem (Richardson [Ric68])

There is a subelementary function of $n+1$ variables, $F(y, y_1, \ldots, y_n)$, for which, as y varies over the natural numbers,

1. *There is no algorithm for deciding whether or not there are real numbers $y_1, \ldots, y_n$ such that $F(y, y_1, \ldots, y_n) = 0$.*
2. *There are real numbers $y_1, \ldots, y_n$ such that $F(y, y_1, \ldots, y_n) \leq 1$ if and only if there are real numbers $y_1, \ldots, y_n$ such that $F(y, y_1, \ldots, y_n) = 0$*

The analytic machinery



The analytic machinery

Theorem

Let $h(w) = w \sin w$ *and* $g(w) = w \sin w^3$*. For every* $y_1, y_2 \in \mathbb{R}$ *and* $\delta \in \mathbb{R}^+$*, there is* $w \in \mathbb{R}^+$ *so that* $|h(w) - y_1| < \delta$ *and* $g(w) = y_2$*.*

Theorem (Generalization, induction)

For every $y_1, y_2, \ldots, y_n \in \mathbb{R}$ *and every* $\delta \in \mathbb{R}^+$*, there is a number* $w \in \mathbb{R}$ *so that*

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h(x) &= x\sin(x) \\
g(x) &= x\sin(x^3) \\
x_1 &= h(x) \\
x_2 &= h \circ g(x) \\
x_3 &= h \circ g \circ g(x) \\
&\cdots \\
x_{n-1} &= h \circ \overbrace{g \circ \ldots \circ g}^{n-2}(x) \\
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$$G[p](m, y) = F[p](m, y_1(w), \ldots, y_n(w))$$

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$$\begin{array}{rcl} \exists y_1, \ldots, y_n \in \mathbb{N}_1 \; p(m, y_1, \ldots, y_n) & = & 0 \\ & \textit{iff} & \\ \exists y \in \mathbb{R} \; G[p](m, y) & \leq & 1 \\ & \textit{iff} & \\ \forall z > 0 \; \exists y \in \mathbb{R} \; G[p](m, y) & \leq & z \end{array}$$

Theorem (Richardson, 1968)

There is a elementary function of two variables, $G(m, y)$, such that, as m varies over $\mathbb{N}_1$, there is no algorithm for deciding whether there is a real number y such that $G(m, y) \leq 1$.

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Richardson's results (1)

Theorem (Richardson [Ric68])

If Φ contains the identity function, the rational numbers, π, the real-valued functions of expressions $|y|$ and $\sin(y)$, and is closed under addition, subtraction, multiplication, and composition, then the identity problem for $(\mathcal{E}, \Phi)$ is undecidable.

Proof:

Take $B(m,y) = |G(m,y)-1| - (G(m,y)-1)$. We have that $\exists y \; G(m,y) < 1$ if and only if $B(m,y) \not\equiv 0$.

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Theorem (Richardson [Ric68])

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If such integration problem were solvable, we would be able to decide, for each $m \in \mathbb{N}_1$, whether there were a function $f \in \Phi$ so that

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Reflection in Physics

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Theorem (Subelementary motion)

There is no general algorithmic procedure to determine whether an arbitrary motion in the $\langle x, y\rangle$-plane, $r(t) = \langle x(t), y(t)\rangle$, will cross the y-axis.

Proof: The reading of rewriting

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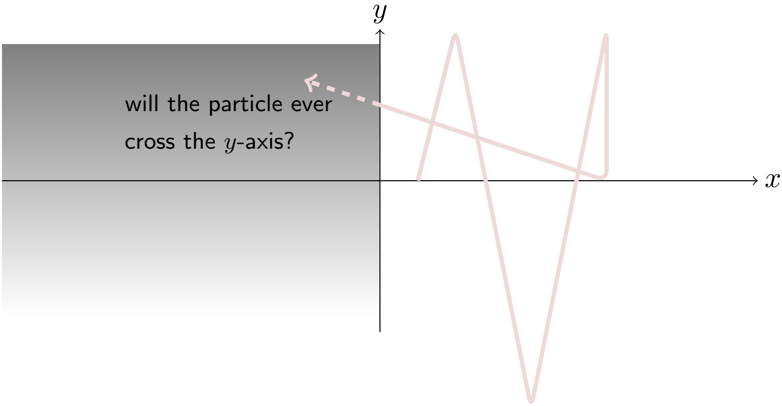

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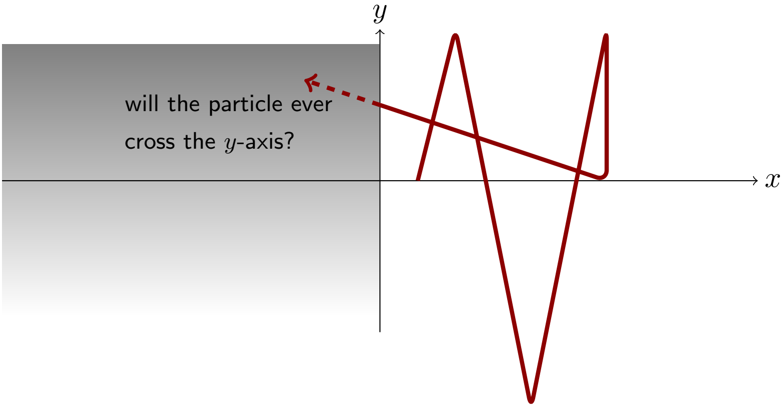
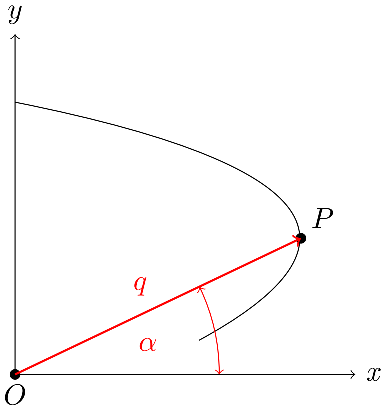


Undecidability in Physics II

Hamiltonians



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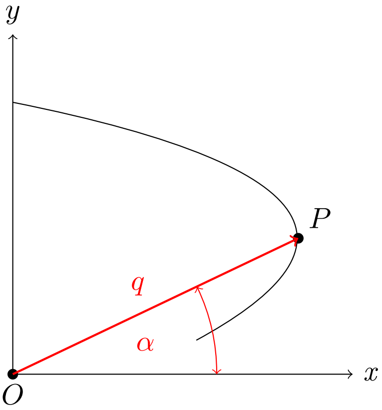

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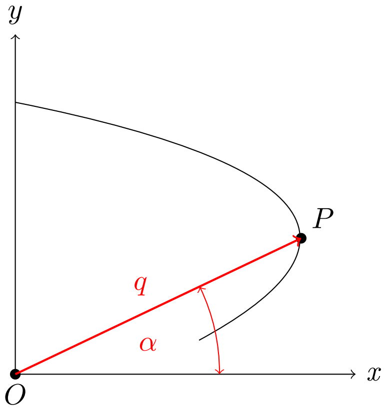

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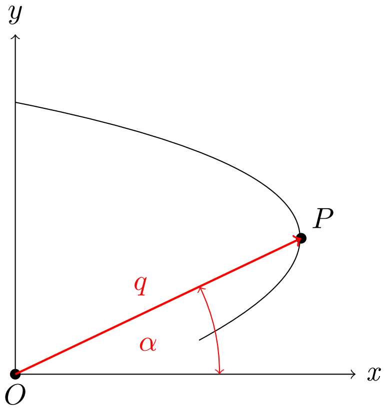

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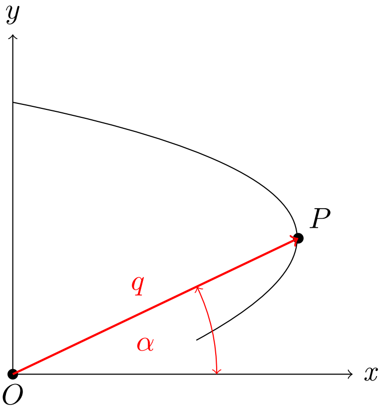

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Undecidability in Physics

A. J. Lichtenberg and M. A. Liberman, *Regular and Stochastic Motion*

Are there general methods to test for the integrability of a given Hamiltonian? The answer, for the moment, is no. We can turn the question around, however, and ask if methods can be found to construct potentials that give rise to integrable Hamiltonians. The answer is that a method exists, at least for restricted class of problems, but the method becomes rapidly very tedious as the forms allowed for the integrals of the motion are expanded.

Hamiltonians

Definition (Poisson brackets)

$$[f, g] = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} \right)$$

If the Poisson bracket of f and g vanishes ($[f, g] = 0$), then f and g are said to be in involution.

Theorem

A Hamiltonian system is completely integrable if and only if the constants of motion are in mutual involution.

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Hamiltonians

Completely integrable systems for the case $H(q, p, t) = H(q, p)$, Francisco Dória and Newton da Costa in [dD91]

If L is a putative constant of motion, then $[H, L] = 0$. E.g., if $H_k = \frac{B(k,q)}{2m} p^2$ and $L = qp_\alpha$, then we will have the trouble

$$\frac{\partial H_k}{\partial q}\frac{\partial L}{\partial p} - \frac{\partial H_k}{\partial p}\frac{\partial L}{\partial q} = -\frac{p}{m}B(k, q)p_\alpha = 0$$

Off to Infinite

Singularidade

Definition (Singularidade)

Uma singularidade é um valor do tempo $t = t^{\star}$ onde deixa de existir solução da equação física que determina a trajetória de um ponto material.

Example (Exemplo e conjetura)

E.g., a colisão é uma singularidade. Mas serão colisões todas as singularidades? Este problema foi levantado na viragem do século XIX para o século XX por Painlevé e von Zeipel. Henri Poincaré conjeturou que havia singularidades sem colisões (Conjetura de Painlevé Poincaré). The way to the solution was provided by Sundman, Wintner, Saari, Mather and McGehee, Anosov, Xia, Gerver.

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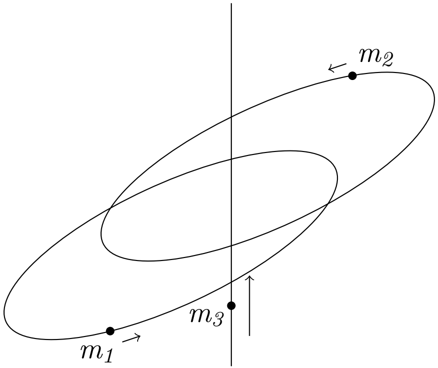

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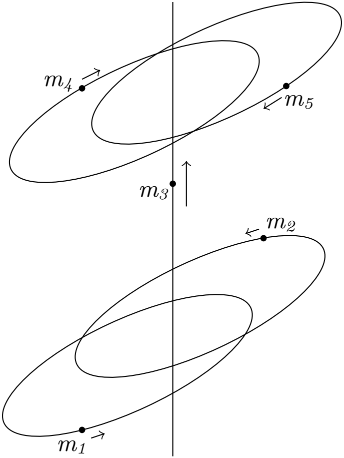
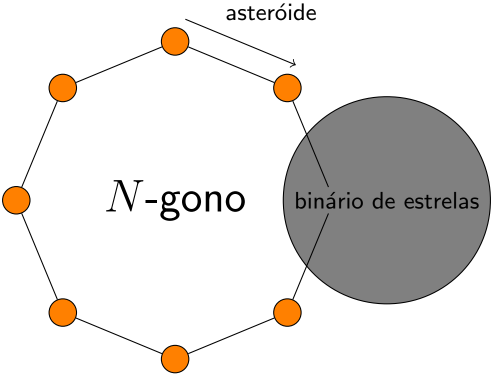
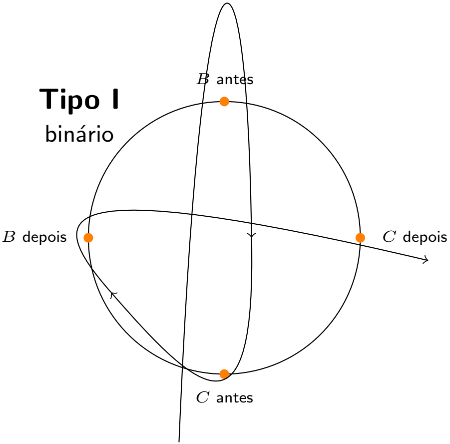


Solução 2-D, Joseph Gerver [Ger91]



Topologia TIPO I



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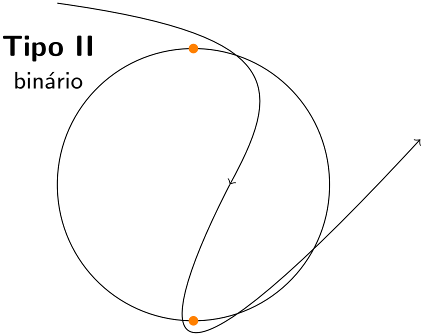

Topologias

Teorema (Topologias incontáveis)

N pontos materiais num plano, cujas massas, posições e velocidades iniciais se encontram num hipercubo de números reais, podem descrever um número não contável de trajetórias distintas em 1 segundo.

Teorema (Limitações das máquinas de Turing)

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Warren Smith e o “computador” Gerver-Smith

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$\mathcal{M}_1$: máquina arbitrária que imprime a sequência dos seus estados, q_0 ou q_1, como sequência binária

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trajetória descrita pela sequência de bits

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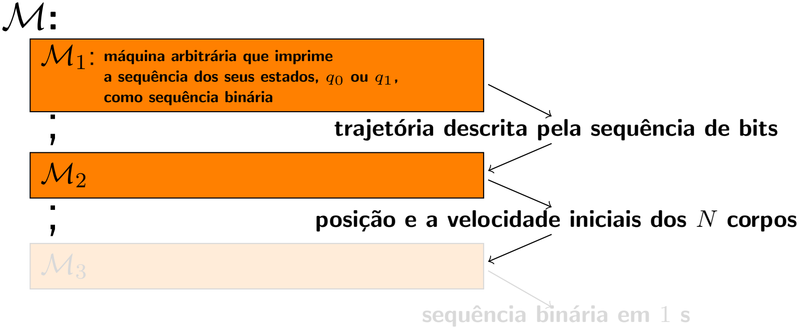
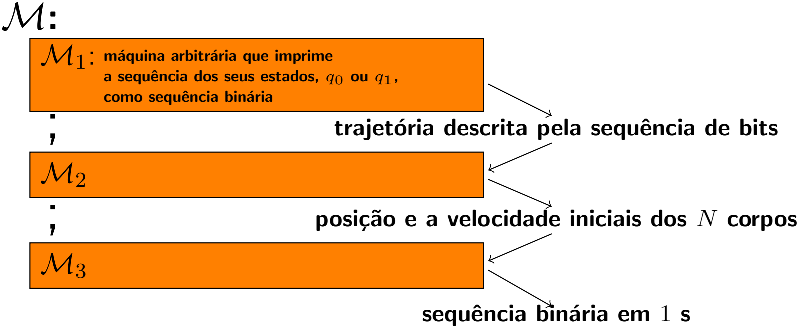


Warren Smith e o “computador” Gerver-Smith



The halting revisited

Description of $\mathcal{M}_3$

Given the initial real number data in such a form that $\mathcal{M}_3$ can access more bits on demand, by some integration scheme, $\mathcal{M}_3$ simulates the motion of the n-body system to sufficient accuracy to be confident it knows the topology of the trajectories the bodies take in $1s$.

Teorema (A decisão da paragem em 1 segundo, Smith [Smi06])

A máquina global, que resulta do processamento paralelo das máquinas $\mathcal{M}_1$, $\mathcal{M}_2$ e $\mathcal{M}_3$, designada por $\mathcal{M} = \mathcal{M}_1 || \mathcal{M}_2 || \mathcal{M}_3$, para se e só se os $2 \times N + 1$ corpos não atingem a singularidade em 1 segundo.

Theorem (Solving the halting problem in $1s$)

$\mathcal{M}_3$ halts if and only if the N bodies do not reach the singularity in $1s$.

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Negative gravity

$$\frac{d\mathbf{p}_i}{dt} = -\sum_{j\neq i} G m_i m_j \frac{r_{ij} - \eta(m_i^{1/3} + m_j^{1/3})}{r_{ij}^3} \frac{\mathbf{r}_{ij}}{r_{ij}}$$

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$\mathbf{r}'_j$ given by the Lorentz transformation

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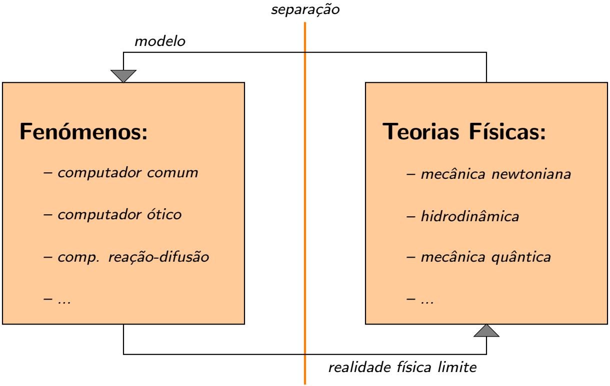
Church-Turing Thesis II

Perspetiva de Kreisel [Kre74, Kre87]



The Church-Turing Thesis

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Church-Turing thesis states that the set of things commonly understood to be computation is identical with the set of tasks that can be carried out by a Turing machine.

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Postulate (As read by Kreisel 1987; statement 1)

Any physical system that purports to be a computer is not capable of any computational task that a Turing machine in incapable of.

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A theory is mechanistic if every sequence of natural numbers or every real number which is well defined (observable) according to theory is recursive or more generally, recursive in the data.

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Every function computable by an abstract human being following a routine procedure is Turing machine computable.

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Abstract of Smith's paper on the n-body system, [Smi06]

Church's thesis is at the foundation of computer science. We point out that any particular set of physical laws, Church's thesis need not merely be postulated, in fact it may be decidable. Trying to do so is valuable. In Newton's laws of physics with point masses, we outline a proof that Church's thesis is false; physics is unsimulable. But with certain more realistic laws of motion, incorporating some relativistic effects, the [...] Church's thesis is true.

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Conclusão: Programa filosófico

Física simulável

Se a construção de Warren Smith tivesse sido apresentada no princípio do século XX, teriam os físicos reformulado a física newtoniana de modo a tornar a física simulável e reestabeler a tese de Church-Turing?

CT como refutação

Poderemos usar o argumento computacional (CT) como refutação de uma teoria científica? Caso contrário, qual é o significado de uma física não simulável?

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