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Why common factors in international bond returns are not so common

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Abstract

This paper analyzes the common factor structure of US, German, and Japanese Government bond returns. Unlike previous studies, we formally take into account the presence of country-specific factors when estimating common factors. We show that the classical approach of running a principal component analysis on a multi-country dataset of bond returns captures both local and common influences and therefore tends to pick too many factors. We conclude that US bond returns share only one common factor with German and Japanese bond returns. This single common factor is associated most notably with changes in the level of domestic term structures. We show that accounting for country-specific factors improves the performance of domestic and international hedging strategies.

JEL Code: F21; G15; E43

Keywords: Bond returns; Factor analysis; Principal component analysis

1. Introduction

Traditional principal component analysis (hereafter PCA) provides much of the intuition for the dynamics of bond yields. Empirical analysis generally determines that three principal components are needed to almost fully explain the dynamics of the term structure of interest rates

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(see Litterman and Scheinkman, 1991; Lardic et al., 2003). The interpretation of these principal components in terms of level, slope, and curvature describes how the yield curve shifts or changes shape in response to a shock on a yield-curve factor. For the postwar period, the first three principal components already capture over 96% of the total variation in US bond yield changes and in the case of bond yield levels, the proportion exceeds 99% (see Piazzesi, 2003, Table 1). Statistical factors extracted from PCA are useful in thinking about the driving forces of the yield curve in finance and macroeconomics. Indeed, latent factors implied by estimated affine term structure models typically behave like the first principal components (see Bams and Schotman, 2003; Dai and Singleton, 2003) and have important macroeconomic and monetary policy underpinnings (see Rudebusch and Wu, 2004). Moreover, the three-factor decomposition of the term structure is rather stable through time (see Bliss, 1997; Chapman and Pearson, 2001; Pérignon and Villa, 2006).

The aim of this paper is to analyze the common factor structure of US, German, and Japanese Government bond returns using PCA and alternative techniques. Understanding the commonalities between different country's term structures is useful for assessing the potential for international diversification and managing the risk of international bond portfolios. To perform this task, we have to model a multi-country covariance matrix that contains (1) the variances of bond returns in each country, (2) the covariances between domestic bond returns with different maturities, and (3) the covariances between bond returns across countries. Two different approaches have been proposed in the literature to study the factors affecting international bond yields or returns. One approach is to estimate the factor structure in each country and then compare the domestic structures based on correlation measures. The other involves jointly studying several domestic term structures and directly extracting the common factors.

In order to study *separately* the factor structures of bond returns in several countries, PCA can be run on each domestic term structure. By applying this technique to the US, German, and

Table 1
Descriptive statistics

	Mean	Std-dev	Skewness	Kurtosis	Rho(l)	ADF
US 1–3 years	6.43	1.70	−0.12	3.71	−0.04	−8.21*
US 3–5 years	7.26	3.63	−0.18	3.52	−0.09	−8.63*
US 5–7 years	7.65	4.87	−0.30	3.92	−0.12	−8.95*
US 7–10 years	7.82	6.25	−0.37	4.21	−0.15	−9.30*
US >10 years	8.50	8.85	−0.32	4.20	−0.15	−9.75*
Germany 1–3 years	6.12	2.84	0.06	5.98	−0.34	−10.41*
Germany 3–5 years	6.95	3.56	0.12	4.11	−0.18	−8.97*
Germany 5–7 years	7.46	4.29	−0.17	6.15	−0.15	−9.27*
Germany 7–10 years	7.34	5.45	−0.71	5.81	−0.05	−9.57*
Germany >10 years	8.28	8.32	−0.43	5.12	−0.07	−9.82*
Japan 1–3 years	7.50	3.07	0.74	9.24	−0.36	−9.91*
Japan 3–5 years	9.80	4.34	0.62	7.02	−0.17	−8.33*
Japan 5–7 years	10.07	5.05	0.25	5.88	−0.13	−8.19*
Japan 7–10 years	10.37	6.19	0.16	5.75	−0.06	−8.28*
Japan >10 years	10.96	7.79	−0.11	5.50	−0.08	−7.84*

Note: Descriptive statistics are computed from the 510 weekly observations, from January 8, 1990 to October 11, 1999, of Government bond returns, with five maturities. Std-dev stands for standard deviation, Rho(l) for autocorrelation, ADF for Augmented Dickey–Fuller test. Mean returns and standard deviations have been annualized. *Indicates that the unit root hypothesis can be rejected at the 1% level, and then that the series is stationary.

Japanese term structures, Driessen et al. (2003) find that the factor loadings in different countries are very similar but that the explained variance per factor is quite different across countries. Alternatively, common PCA can be used to extract the main factors in each country. This parsimonious approach developed by Flury (1988) is consistent with the above stylized fact since it assumes that the eigenvectors in several groups are the same, whereas the eigenvalues vary among groups. Furthermore, combining information from several samples leads in general to more stable estimates. However, since common PCA neglects the covariations between bond returns across countries, it fails to estimate common factors.

In order to study *jointly* the term structures of interest rates in several countries, PCA can be run on a pooled dataset of several domestic term structures. Following this approach, Rodrigues (1997) concludes that the three-factor structure does not adequately describe the common movements among several domestic term structures and that more factors are required. In the same vein, Driessen et al. (2003) conclude that a five-factor model explains almost perfectly the dynamics of Government bonds returns from US, Germany, and Japan. Using 10 years of weekly data of LIBOR/swap rates for US dollar and Japanese yen, Leippold and Wu (2003) need five principal components to capture almost all variations in the system. Therefore, there is an inconsistency between the large number of factors suggested by statistical analyses and the single common factor used in affine term structure models in an international setting (see Backus et al., 2001; Han and Hammond, 2003; Ahn, 2004). Indeed, the basic assumption in these models is that each domestic term structure is driven by one specific factor in each country and one factor that is common to both countries.[1]

Although PCA has been broadly used to estimate the key factors driving a single domestic term structure, one may critique its application in a multi-country setting.[2] Indeed, the goal of PCA is to extract the factors that maximize the explained variance, but not necessarily factors that are common across countries. For this reason, running a PCA on a dataset pooling different domestic term structures does not prevent selecting a local factor − whose influence is limited, by definition, to a single country − with a huge variance as a common factor. Consequently, one never knows whether a given principal component is associated with a common or a local factor.[3] In the case of two countries, the inter-battery factor analysis (hereafter IBFA) offers an alternative to PCA. This multivariate statistical technique developed by Tucker (1958) has been widely used in psychology but only sparingly in finance − the only two applications are Cho (1984) and Cho et al. (1986) that both analyze stock returns. Unlike PCA, IBFA intends to capture all comovements across domestic term structures by first estimating common factors. Once the influence of the common factors has been discarded, the two groups of securities become independent, i.e., the residual covariance matrix is block-diagonal. Each country-specific residual covariance matrix may still contain factors that affect only securities within each country.

In the empirical analysis, we apply our methodology to US, German, and Japanese Government bond returns over the 1990s. Using IBFA, we study the number and the nature of the common factors driving international bond returns. We conclude that US bond returns

[1] In the single-country case, such an inconsistency does not arise. Indeed, both PCA and affine term structure models conclude that three factors are required to model a given domestic term structure of interest rates (see Piazzesi, 2003).

[2] Lekkos (2000) also criticizes the use of PCA in a domestic setting. He claims that the explanatory power of the first factor is amplified by the no-arbitrage restrictions linking bond yields and forward rates.

[3] When controlling for local factors, Barr and Priestley (2004) find that the average contribution of world factors to bond returns is only 70%. This figure is significantly lower than the total variance explained by the first five principal components of the multi-country dataset, i.e., over 95%, in Driessen et al. (2003).

share only one common factor with, respectively, German and Japanese bond returns. We find that the single common factor is associated most notably with changes in the level of domestic term structures. We also extend the standard methodology to the case of more than two countries and allow for time-variation in the covariance matrix of bond returns, while maintaining the spirit of IBFA. As an illustration, we present a factor-based strategy to hedge long-term bonds using domestic or foreign shorter-term bonds. We find that accounting for country-specific factors improves the performance of hedging strategies.

The remainder of the paper is organized as follows. Section 2 details the methodology, Section 3 presents the empirical analysis, while Section 4 concludes.

2. Factor decomposition of international bond returns

2.1. Model specification

Let R_t be the $2M$-vector containing the bond returns in two countries measured in excess of the risk-free interest rate at time t. In each country, we observe the returns on bonds of M maturities at time $t = 1, \ldots, T$ and $R'_t = [R'_{1t}, R'_{2t}]$. We take the viewpoint of a country-one investor (domestic investor) and then convert country-two returns into country-one currency returns. We consider bond positions that are hedged for currency risk and use the country-one risk-free interest rate to compute the excess returns in both countries.

We denote by Σ_{kk} the $M \times M$ covariance matrix of the excess hedged bond returns in country k, Σ_{kl} the $M \times M$ cross-covariance matrix of the excess hedged bond returns between countries k and l, and Σ the $2M \times 2M$ overall covariance matrix. We assume that excess hedged bond returns are linearly related to c common factors contained in vector F_t:

$$R_{kt} = E(R_k) + B_k F_t + \epsilon_{kt}, \quad k = 1, 2, \tag{1}$$

where the B_k matrix contains the loadings on the common factors in country k and ϵ_k is the vector of residuals in country k. We assume that F_t and ϵ_{kt} follow a multivariate normal distribution with zero means and are orthogonal to each other. It is also convenient to assume that the common factors are orthogonal, i.e., $E(FF') = I_c$. These assumptions imply that R follows a multivariate normal distribution with mean $E(R)$ and covariance matrix:

$$\Sigma = BB' + \Psi, \tag{2}$$

where $B' = [B'_1, B'_2]$. The only difference between the present model and a standard factor analysis comes from the potential presence of additional factors being common to one country only. Consequently, the Ψ matrix is not assumed to be diagonal, but block-diagonal:

$$\Psi = \begin{bmatrix} \Psi_{11} & 0 \\ 0 & \Psi_{22} \end{bmatrix}, \tag{3}$$

where Ψ_{kk} are non-negative definite matrices. As the diagonality of the residual covariance matrix is not necessary for the arbitrage pricing theory (APT) of Ross (1976) to be valid (see Chamberlain and Rothschild, 1983), this model fits into the APT framework.

2.2. Estimation of the inter-battery factor analysis model

The model presented in Eqs. (1)–(3) is an IBFA model (see Tucker, 1958) and can be estimated by maximum likelihood following Browne (1979). If S is the usual unbiased estimate of Σ obtained from a sample of T independent observations on R, then $(T-1)S$ has a Wishart distribution with $(T-1)$ degrees of freedom. Browne (1979) shows that the maximum likelihood estimates $\widehat{B}' = [\widehat{B}'_1, \widehat{B}'_2]$ and $\widehat{\Psi}$ must satisfy the following equations:

$$\widehat{B}_2 = S_{21}S_{11}^{-1}\widehat{B}_1\left(\widehat{B}'_1S_{11}^{-1}\widehat{B}_1\right)^{-1}, \tag{4}$$

$$S_{12}S_{22}^{-1}S_{21}S_{11}^{-1}\widehat{B}_1 = \widehat{B}_1\left(\widehat{B}'_2S_{22}^{-1}\widehat{B}_2\right)\left(\widehat{B}'_1S_{11}^{-1}\widehat{B}_1\right), \tag{5}$$

$$\Psi = \mathrm{Bdiag}\left(S - \widehat{B}\widehat{B}'\right), \tag{6}$$

where S_{kl} is the sample cross-covariance matrix between countries k and l and Bdiag(E) represents a block-diagonal matrix formed from the principal submatrices of E.

To simultaneously evaluate whether the variables are generated by an IBFA model and find the appropriate number of common factors, one has to investigate if there exists a "sufficiently small" c such that the covariance matrix can be written as $BB' + \Psi$. If c equals 0, then no common factor structure exists. If no c can be found, then again the factor model does not hold: in this case the number of factors needed to explain the variability of the observable variables is too high. If a small value of c is found, then the model is not rejected and, at the same time, the dimension of the factor structure is obtained. In order to test the fit of the model, the following statistic can be used:

$$-2\ln\Delta = -(T-1)\ln\prod_{j=c+1}^{M}\left(1-\widehat{\rho}_j^2\right), \tag{7}$$

where $\widehat{\rho}_j^2$ are the eigenvalues of $S_{12}S_{22}^{-1}S_{21}S_{11}^{-1}$ and Δ is the likelihood ratio. The limiting distribution of $-2\ln\Delta$ is chi-square with $(M-c)^2$ degrees of freedom (see Browne, 1979; Cho, 1984).

2.3. Comparing inter-battery factor analysis and principal component analysis

Driessen et al. (2003) have proposed a related model for international bond returns, which is called the principal component model. They claim that the common factors driving international bond returns can be estimated by the principal components of the overall covariance matrix Σ. In order to see the main differences between the principal component and IBFA models, let $p = 2M$. Recall also that PCA breaks down the covariance matrix Σ as:

$$\Sigma = A\Lambda A', \tag{8}$$

where A is a $p\times p$ orthogonal matrix containing the p eigenvectors and Λ is a $p\times p$ diagonal matrix containing the p eigenvalues. We partition the A matrix as $[A_1, A_2]$ where A_1 is a $p\times c$

matrix. Let Λ_1 be the $c \times c$ upper left-hand corner matrix of Λ and Λ_2 be the lower right-hand corner matrix of Λ. Defining $X = R - E(R)$, we can write:

$$X = AA'X = A_1 Z + U, \tag{9}$$

where $Z = A_1'X$ and $U = A_2 A_2' X$.

An associated model can be obtained by letting $B = A_1 \Lambda_1^{1/2}$ and $F = \Lambda_1^{-1/2} Z = \Lambda_1^{-1/2} A_1' X$ so that $X = BF + U$ which remarkably resembles the IBFA model. As in the IBFA model, the factors have an identity covariance matrix and are uncorrelated with the residuals:

$$E(FF') = E\left(\Lambda_1^{-1/2} Z (\Lambda_1^{-1/2} Z)'\right) = \Lambda_1^{-1/2} E(ZZ') \Lambda_1^{-1/2} = I_c \tag{10}$$

$$E(FU') = E\left(\Lambda_1^{-1/2} A_1' X \left(A_2 A_2' X\right)'\right) = \Lambda_1^{-1/2} A_1' \Sigma A_2 A_2' = 0. \tag{11}$$

However, the principal component model does not account for country-specific factors and its residual covariance matrix is not necessarily block-diagonal:

$$E(UU') = E\left(A_2 A_2' X \left(A_2 A_2' X\right)'\right) = A_2 \Lambda_2 A_2'. \tag{12}$$

Therefore, the common factors obtained from PCA do not necessarily explain all the comovements between the two groups of securities. Since PCA is not primarily concerned with the estimation of common factors, but with the estimation of factors that maximize the explained variance, other principal components have to be added to explain the covariability among international bond returns. In conclusion, the main drawback of PCA in the presence of several groups is the fact that estimated factors jointly capture both local and common influences.

3. Empirical analysis

3.1. Data

Our dataset consists of total weekly returns on Merrill Lynch Government Bond Indices for the US, Germany, and Japan, from January 8, 1990 to October 11, 1999.[4] For each country, the bond index is denominated in US dollars and five maturity classes are available: 1–3 years, 3–5 years, 5–7 years, 7–10 years, and more than 10 years. Hence, five time-series of 510 observations are obtained. In Table 1, we provide some descriptive statistics for the hedged bond returns.[5] It appears that both the level and the volatility of bond returns increase with maturity. According to the skewness and the kurtosis figures, the distribution of the bond returns is slightly asymmetric and leptokurtic. Moreover, bond return time-series exhibit stationarity and low autocorrelation. In the following, we use excess hedged bond returns, defined in excess of the 1-week eurodollar interest rate.

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[4] To ease the comparison, we use the same data and apply the same hedging procedure as Driessen et al. (2003). We warmly thank Joost Driessen for providing us with the data.

[5] German and Japanese bond index returns have been hedged for currency risk using data on spot and forward exchange rates: $R_{t,m}^{\text{hedged}} = R_{t,m} + \ln(S_t) - \ln(F_{t,1\ \text{week}})$, where $R_{t,m}$ is the raw bond return, $R_{t,m}^{\text{hedged}}$ the hedged bond return in dollars, m the maturity of the bond index, S_t the spot exchange rate, and $F_{t,1\ \text{week}}$ is the one-week forward exchange rate. Both raw and hedged returns are continuously compounded.

3.2. Single-country analysis

Panel A of Table 2 presents the results of a separate PCA run for each country. The first three factors explain 99.9% of the variation in bond returns for the US, and 98.7% for Germany and Japan. The first factor accounts for most of the variability in bond returns: 96.9% for the US, 84.9% for Germany, and 89.7% for Japan. As the loadings on the first factor are always positive, the first factor captures changes in the level of the term structure. We interpret the second factor as a slope factor since its loadings increase uniformly from a relatively large negative value for short maturities to a positive value for the longest maturities. We interpret the third factor as a curvature factor since its loadings are negative for short and long maturities and positive for intermediate maturities. Fig. 1 plots the values of the first three factor loadings for bond returns estimated by running a PCA on each domestic term structure.

As pointed out by Driessen et al. (2003), the factor loadings show a consistent pattern across countries but the explained variance per factor varies from one country to another (see eigenvalues in Table 2). The common PCA appears particularly adapted to this situation since it

Table 2
First three eigenvectors and eigenvalues for the US, Germany and Japan

Panel A: Separate principal component analyses

	US			Germany			Japan		
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3
1–3 years	0.122	−0.315	−0.527	0.181	−0.428	−0.541	0.199	−0.492	−0.571
3–5 years	0.280	−0.507	−0.423	0.281	−0.396	−0.304	0.337	−0.485	−0.116
5–7 years	0.388	−0.429	0.122	0.361	−0.397	−0.030	0.417	−0.299	0.109
7–10 years	0.504	−0.218	0.669	0.471	−0.296	0.752	0.517	−0.061	0.699
>10 years	0.709	0.643	−0.284	0.732	0.644	−0.218	0.637	0.655	−0.400
Eigenvalues	2.916	0.080	0.009	2.249	0.286	0.079	2.635	0.210	0.055
% Variance	0.969	0.027	0.003	0.849	0.108	0.030	0.897	0.071	0.019

Panel B: Common principal component analysis

	Eigenvector 1	Eigenvector 2	Eigenvector 3
1–3 years	0.137	−0.376	−0.545
3–5 years	0.292	−0.495	−0.360
5–7 years	0.393	−0.386	0.116
7–10 years	0.506	−0.207	0.684
>10 years	0.697	0.649	−0.303
Eigenvalues US	2.915	0.081	0.009
% Variance	0.969	0.027	0.003
Eigenvalues GE	2.237	0.283	0.086
% Variance	0.844	0.107	0.032
Eigenvalues JP	2.611	0.227	0.059
% Variance	0.888	0.077	0.020

Note: Panel A displays the factor loadings on the first three factors obtained from separate principal component analyses for the US, German (GE), and Japanese (JP) term structure of bond returns. E_1, E_2 and E_3 are the eigenvalues associated with the first three eigenvectors for each country. Panel B displays the factor loadings on the first three factors obtained from a common principal component analysis run jointly on the three domestic term structures of bond returns. In both panels, the first column indicates the five available maturities for the bond indices, and % variance indicates the percentage of the variability of the original data captured by each factor.

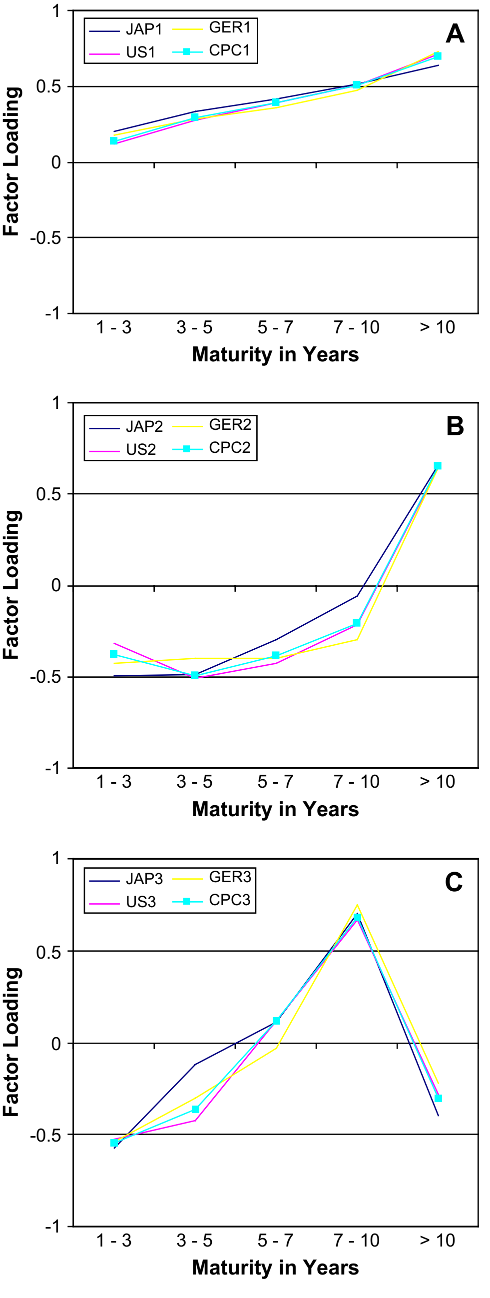

Fig. 1. Factor loading on the first three factors. Note: Panels A, B and C display the factor loadings on the first three factors. The following abbreviations are used in these panels: JAP1, JAP2 and JAP3 are the factor loadings obtained from a separate principal component analysis run on the Japanese term structure, US1, US2 and US3 are the factor loadings obtained from a separate principal component analysis run on the US term structure and GER1, GER2 and GER3 are the factor loadings obtained from a separate principal component analysis run on the German term structure, and CPC1, CPC2 and CPC3 are the factors loading obtained from a common principal component analysis jointly run on the three domestic term structures of bond returns.

assumes that the eigenvectors in the considered countries are the same, whereas eigenvalues may vary across countries. This factor extraction method estimates factors with exactly the same loadings in all countries. Common PCA is clearly related to PCA since it decomposes each country-level covariance matrix as:

$$\Sigma_{kk} = A\Lambda_k A', \quad k = 1,\ldots,K. \tag{13}$$

The ith column of the $M \times M$ matrix A gives the coefficients of the ith principal component, and the diagonal elements of Λ_k, give the variances of the principal components in country k. Common principal components in country k are given by $A'R_k$. Technically, common PCA is a joint-diagonalization of the three domestic covariance matrices (see Appendix 1). We notice in Fig. 1 and in Panel B of Table 2 that the factor loadings obtained under the common PCA assumption are quite close to those obtained individually for each country. The cost of the joint-diagonalization of the three covariance matrices is a slight decrease in the three-factor model performance: 99.9% for the US, 98.3% for Germany, and 98.5% for Japan.

Comovements among domestic bond markets are investigated through correlation coefficients between the common principal components in the three countries.[6] The first US factor covaries more strongly with the first German factor (correlation coefficient of 0.401) than with the first Japanese factor (0.151). Moreover, the correlation between the first US factor and the second German factor (0.216) is more important than the correlation between the first US and Japanese factors. Other correlation coefficients are significantly smaller.

3.3. Multi-country analysis

We conduct an IBFA to estimate the common factors driving international hedged excess bond returns. As bond returns have been computed from indices denominated in US dollars and currency risk has been hedged away, we implicitly adopt the viewpoint of a US investor. We then successively study the common structure among US and German bond returns and among US and Japanese bond returns.[7]

The main results of the IBFA are summarized in Table 3. We find that, for the two samples, only one factor can be considered as common as indicated by the statistical test in Eq. (7). For US and German bonds and for US and Japanese bonds, we can reject the assumption that there is no common factor at the 5% confidence level ($p\text{-value}_{\text{US-Germany}} = 0.000$ and $p\text{-value}_{\text{US-Japan}} = 0.001$) but we cannot reject the assumption that there is only one common factor at the 5% confidence level ($p\text{-value}_{\text{US-Germany}} = 0.392$ and $p\text{-value}_{\text{US-Japan}} = 0.095$). This result is consistent with the conclusion of Ilmanen (1995) that a single global risk factor is sufficient to explain international bond returns. However, it differs from the conclusion of Driessen et al. (2003) who claim, based on a PCA, that five common factors jointly determine international bond returns.

To empirically compare IBFA and PCA, we report in Table 4 the residual covariance matrices in each sample. If all common factors were correctly estimated then these matrices should be block-diagonal. Interestingly, when a single common factor is estimated by IBFA, the

[6] Note that, unlike Rodrigues (1997), we do not investigate the covariations across principal components estimated from a separate PCA in each country. Indeed, comparing components estimated from several PCA may be less meaningful since the factor loadings are not identical in different countries.

[7] A joint analysis of the common structure of US, German, and Japanese bond returns is proposed in Section 3.4.

Table 3
Test for the number of common factors

Number	$-2 \ln \Delta$	df	p-Value	χ^2 ($\alpha = 5\%$)
Panel A: US–Germany				
0	172.405	25	0.000	37.653
1	16.909	16	0.392	26.296
2	6.719	9	0.666	16.919
3	2.549	4	0.636	9.488
4	0.010	1	0.922	3.842
Panel B: US–Japan				
0	54.047	25	0.001	37.653
1	23.750	16	0.095	26.296
2	10.056	9	0.346	16.919
3	2.717	4	0.606	9.488
4	0.047	1	0.829	3.842

Note: This table displays the results of the statistical test that permits to simultaneously (1) evaluate whether the bond returns are generated by an inter-battery factor analysis model and (2) find the appropriate number of common factors (in bold) for the US and Germany (Panel A) and the US and Japan (Panel B). The statistic $-2 \ln \Delta$ is equal to $= -(T-1)\ln \prod_{j=c+1}^{M}(1-\widehat{\rho}_j^2)$. The appropriate number of common factors is the smallest number of factors for which the overall covariance matrix can be written as $\Sigma = BB' + \Psi$ where Ψ is a block-diagonal residual covariance matrix and B is the matrix of the loadings on the common factors. The columns indicate, respectively, the number of common factors, the value of the statistic where Δ denotes the likelihood ratio, the number of degrees of freedom, the associated p-value, and the critical value of a chi-square distribution at the 5% confidence level.

residual covariance matrices appear to be block-diagonal. This result indicates that most of the comovements between bond returns across countries can be captured by a single common factor. On the other hand, when the first principal component is used as the common factor, the residual covariance matrices are far from being block-diagonal. These results indicate that while the primary concern of PCA is the estimation of factors that maximize the explained variance, these factors are not necessarily common.

We now turn to the interpretation of the common factor. Driessen et al. (2003) regress each of their five common factors on the three country-specific factors, which are estimated by running separate PCA. In this paper, we perform an IBFA directly on the common principal components. We then use the estimated loadings to interpret the multi-country factors in terms of country-specific factors. Note that we use the all five standardized common principal components in each country since a common principal component that explains a minor part of bond return covariations within a specific country may explain a large part of bond return covariations between two countries. In both samples, we find that the single common factor is associated most notably with changes in the level of domestic term structures (see Table 5).

Finally, we investigate the country-specific or local factors in each country. Since all common influences have been captured by the common factor, country-specific factors can be estimated from the residual covariance matrices. By using a standard PCA, we extract factors that only affect securities within each country. Table 6 presents the variance captured by one common factor and one country-specific factor for each individual excess bond return series. The explained variance is defined as one minus the ratio between the residual variance and the total variance. We first remark that the explained variance is an increasing function of the maturity of the bonds. The two-factor model, with one common factor and one country-specific factor, provides a good fit of the average bond returns, as measured by the explained variance for each

Table 4

Residual covariance matrices

Panel A: US–Germany

Inter-battery factor analysis

0.0368	0.0710	0.0866	0.1015	0.1207					
0.0710	0.1483	0.1869	0.2241	0.2743					
0.0866	0.1869	0.2456	0.3015	0.3814					
0.1015	0.2241	0.3015	0.3842	0.5032					
0.1207	0.2743	0.3814	0.5032	0.7413					
0.0029	0.0022	0.0009	−0.0002	0.0070	0.1538	0.1661	0.1876	0.1842	0.2072
0.0038	0.0039	0.0022	0.0013	0.0078	0.1661	0.2182	0.2260	0.2461	0.2870
0.0043	0.0042	0.0031	0.0014	0.0072	0.1876	0.2260	0.2957	0.3061	0.3594
0.0022	0.0010	−0.0001	−0.0013	0.0066	0.1842	0.2461	0.3061	0.3856	0.4181
0.0066	0.0072	0.0046	0.0028	0.0122	0.2072	0.2870	0.3594	0.4181	0.8744

Principal component analysis

0.0194	0.0331	0.0355	0.0362	0.0297					
0.0331	0.0656	0.0759	0.0826	0.0776					
0.0355	0.0759	0.0970	0.1121	0.1186					
0.0362	0.0826	0.1121	0.1430	0.1689					
0.0297	0.0776	0.1186	0.1689	0.2786					
−0.0191	−0.0479	−0.0684	−0.0900	−0.1202	0.1351	0.1325	0.1430	0.1217	0.1087
−0.0265	−0.0635	−0.0895	−0.1167	−0.1576	0.1325	0.1677	0.1618	0.1650	0.1588
−0.0333	−0.0788	−0.1094	−0.1428	−0.1943	0.1430	0.1618	0.2151	0.2074	0.2031
−0.0423	−0.0953	−0.1287	−0.1648	−0.2197	0.1217	0.1650	0.2074	0.2749	0.2425
−0.0639	−0.1453	−0.1993	−0.2565	−0.3469	0.1087	0.1588	0.2031	0.2425	0.5958

Panel B: US–Japan

Inter-battery factor analysis

0.0491	0.0993	0.1270	0.1547	0.1987					
0.0993	0.2121	0.2783	0.3445	0.4511					
0.1270	0.2783	0.3765	0.4739	0.6345					
0.1547	0.3445	0.4739	0.6113	0.8366					
0.1987	0.4511	0.6345	0.8366	1.2306					
−0.0019	−0.0028	−0.0011	−0.0024	0.0116	0.1809	0.2223	0.2405	0.2585	0.2745
−0.0001	0.0001	0.0025	0.0018	0.0216	0.2223	0.3446	0.3738	0.4230	0.4513
0.0015	0.0028	0.0073	0.0085	0.0315	0.2405	0.3738	0.4643	0.5247	0.5910
0.0008	0.0017	0.0065	0.0078	0.0321	0.2585	0.4230	0.5247	0.6627	0.7367
−0.0034	−0.0050	−0.0010	−0.0019	0.0161	0.2745	0.4513	0.5910	0.7367	1.0142

Principal component analysis

0.0247	0.0442	0.0504	0.0556	0.0568					
0.0442	0.0889	0.1072	0.1234	0.1343					
0.0504	0.1072	0.1389	0.1669	0.1946					
0.0556	0.1234	0.1669	0.2146	0.2682					
0.0568	0.1343	0.1946	0.2682	0.4162					
−0.0313	−0.0711	−0.0965	−0.1263	−0.1651	0.1505	0.1644	0.1681	0.1657	0.1596
−0.0490	−0.1102	−0.1509	−0.1968	−0.2626	0.1644	0.2472	0.2520	0.2740	0.2730
−0.0597	−0.1354	−0.1848	−0.2401	−0.3244	0.1681	0.2520	0.3118	0.3382	0.3677
−0.0735	−0.1642	−0.2239	−0.2899	−0.3945	0.1657	0.2740	0.3382	0.4393	0.4732
−0.0920	−0.2011	−0.2730	−0.3530	−0.4874	0.1596	0.2730	0.3677	0.4732	0.7068

Note: These tables display the residual covariance matrices for US and Germany (Panel A) and US and Japan (Panel B). Each residual covariance matrix is estimated using one common factor estimated either using an inter-battery factor analysis or a principal component analysis. If all common factors were correctly estimated the residual covariance matrices should be block-diagonal.

Table 5
Loadings on the single common factor

	CPC_1	CPC_2	CPC_3	CPC_4	CPC_5
Panel A: US–Germany					
US	2.0609	−0.0159	0.0012	0.0000	−0.0003
Germany	1.2502	−0.0874	0.0123	−0.0042	0.0028
Panel B: US–Japan					
US	1.2477	−0.0080	0.0015	−0.0001	−0.0004
Japan	0.8424	−0.0312	0.0085	−0.0051	−0.0007

Note: This table displays the estimated loadings on the common factor estimated using an inter-battery factor analysis. In order to interpret the common factor in terms of domestic factors, the analysis has been performed on the five standardized common principal components (CPC_i, $i = 1, 2, 3, 4, 5$).

sample (92.08% for US–Germany and 93.68% for US–Japan). The performance of our model is slightly better than a two-factor principal component model (91.61% for US–Germany and 93.40% for US–Japan). This empirical evidence supports the assumption made in affine models in an international setting that domestic term structures are driven by one common factor and one country-specific factor.

3.4. Common factors with more than two countries

Existing factor extraction methods have both advantages and limitations when applied to multi-country datasets. In particular, PCA can handle datasets with more than two countries but fails to disentangle common and local influences. On the other hand, IBFA accounts for the presence of local factors when estimating common factors but cannot be applied to more than two countries at the same time. Below, we propose a methodology that permits common factors to be extracted from bond returns in more than two countries, while allowing for local factors.[8] We refer to this model as the multi-country common factor analysis.

We present the model for the special case of three countries.[9] The only parameters that need to be estimated are the loadings on the common factor, B. The residual covariance matrix of country k is denoted by $\Psi_{kk}(B_k) = E(\epsilon_k \epsilon_k')$ and can be expressed as:

$$\Psi_{kk}(B_k) = S_{kk} - B_k B_k', \tag{14}$$

where S_{kk} is the sample covariance matrix of R_k. The overall residual covariance matrix is block-diagonal:

$$\Psi(B) = \begin{bmatrix} \Psi_{11}(B_1) & 0 & 0 \\ 0 & \Psi_{22}(B_2) & 0 \\ 0 & 0 & \Psi_{33}(B_3) \end{bmatrix}. \tag{15}$$

[8] We thank a referee for suggesting that we extend IBFA in the heteroscedastic case and in the case of more than two countries. We present these extensions in Sections 3.4 and 3.5.

[9] This model, as well as the one proposed in the next section, can straightforwardly be extended to more than three countries.

Table 6
Variance explained by two factors

	Inter-battery factor analysis		Principal component analysis	
	US–Germany	US–Japan	US–Germany	US–Japan
US 1–3	0.8031	0.7990	0.7921	0.7950
US 3–5	0.9145	0.9105	0.9093	0.9106
US 5–7	0.9669	0.9652	0.9647	0.9653
US 7–10	0.9897	0.9892	0.9889	0.9893
US >10	0.9766	0.9773	0.9769	0.9773
Germany 1–3	0.6399	–	0.5508	–
Germany 3–5	0.7855	–	0.7558	–
Germany 5–7	0.8623	–	0.8457	–
Germany 7–10	0.8848	–	0.8787	–
Germany >10	0.8977	–	0.9030	–
Japan 1–3	–	0.6344	–	0.5878
Japan 3–5	–	0.8384	–	0.8304
Japan 5–7	–	0.9392	–	0.9323
Japan 7–10	–	0.9562	–	0.9559
Japan >10	–	0.9162	–	0.9151
Total	0.9208	0.9368	0.9161	0.9340

Note: This table displays the variance captured by two factors. In the column headed "Inter-battery factor analysis", we use one common factor extracted using an inter-battery factor analysis and one country-specific factor obtained by running a principal component analysis on each country-specific residual covariance matrix. In the column headed "Principal component analysis", the two factors are the first two principal components of the overall covariance matrix for each pair of countries. We present the explained variance for each individual bond return series and for each sample (US–Germany and US–Japan).

We ensure that each residual covariance matrix is nonnegative definite by using the algorithm of Sharapov (1997).[10] The overall covariance matrix is then given by $\Sigma = BB' + \Psi(B)$. This model preserves the intuition of IBFA since the covariance between bond returns in different countries is only determined by the loadings on the common factor. The covariance between bond returns in the same country also depends on the unmodeled residual factor structure within that country.

We numerically estimate the parameters in B by maximizing the log-likelihood of bond returns:

$$\mathrm{LL}(B;R) = \sum_{t=1}^{T} \log f(R_t; E(R), BB' + \Psi(B)), \tag{16}$$

where $f(\cdot;\ M,\ C)$ is the probability density function of a multivariate normal random variable with mean M and covariance matrix C. In our application, we consider only one common factor although the model can accommodate multi-factor structures. As in IBFA, we extract in a second step the country-specific factor from the residual covariance matrices using PCA.

[10] This algorithm finds the positive-definite matrix which is "closest" to some indefinite matrix but has the same strictly positive principal diagonal. Distance is measured using the Frobenius matrix norm. This algorithm has been used in a related context by Ledoit et al. (2003) to estimate multivariate GARCH models.

We report the estimated loadings on the common factor (15 parameters) and the country-specific factors (15 parameters) in Table 7. We also display the loadings on the first two factors estimated by PCA and IBFA. The most striking result in the table is the remarkable similarity between the IBFA and multi-country common factor analysis loadings. It appears that the dual nature of IBFA is not a serious limitation. The estimated loadings are positive and increase with the bond maturities. However, there are some interesting variations in the results. The US loadings on the common factor are higher when estimated using multi-country common factor analysis than the US–Japan IBFA loadings and lower than the US–Germany IBFA loadings. The results of multi-country common factor analysis are thus a hybrid of the two IBFA results. Moreover, the loadings of Japanese bonds on the common factor are moderately lower than their IBFA counterparts. It appears that the relationship between US and German bonds is most relevant in defining the common factor structure.

3.5. Time-varying covariance matrix

All the factor extraction techniques previously used in this paper maintain the assumption that the covariance matrix of bond returns is constant through time. We now present a method, which we call GARCH common factor analysis, that allows for time-variation in the covariance matrix of bond returns. Furthermore, it is flexible enough to model the common factor structure of more than two countries and to estimate common and country-specific factors.

Table 7
Estimated factor loadings

	IBFA US–Germany		IBFA US–Japan		PCA		MCFA		GARCH CFA	
	F_1	F_2	F_1	F_2	F_1	F_2	F_1	F_2	F_1	F_2
US 1–3	0.134	0.161	0.074	0.195	0.166	−0.102	0.119	0.171	0.180	0.143
US 3–5	0.321	0.356	0.198	0.435	0.385	−0.225	0.289	0.382	0.429	0.259
US 5–7	0.458	0.480	0.281	0.601	0.537	−0.305	0.412	0.519	0.624	0.263
US 7–10	0.604	0.614	0.371	0.777	0.699	−0.395	0.543	0.667	0.826	0.237
US >10	0.873	0.840	0.523	1.094	1.003	−0.535	0.784	0.922	1.240	0.166
Germany 1–3	0.032	0.313	–	–	0.166	0.074	0.035	0.320	0.027	0.346
Germany 3–5	0.161	0.407	–	–	0.298	0.049	0.182	0.398	0.102	0.477
Germany 5–7	0.242	0.497	–	–	0.389	0.023	0.268	0.483	0.174	0.546
Germany 7–10	0.431	0.566	–	–	0.552	−0.023	0.482	0.525	0.300	0.597
Germany >10	0.675	0.859	–	–	0.824	−0.158	0.704	0.834	0.468	0.761
Japan 1–3	–	–	0.009	0.339	0.162	0.287	0.010	0.328	0.006	0.345
Japan 3–5	–	–	0.133	0.535	0.303	0.455	0.097	0.540	0.064	0.587
Japan 5–7	–	–	0.166	0.659	0.371	0.558	0.118	0.666	0.087	0.691
Japan 7–10	–	–	0.277	0.794	0.480	0.676	0.201	0.816	0.128	0.798
Japan >10	–	–	0.395	0.957	0.567	0.831	0.223	1.009	0.128	0.857

Note: This table displays estimates of the loadings on the first two factors (F_1 and F_2) for bond returns in US, Germany, and Japan with maturities in 1–3 years, 3–5 years, 5–7 years, 7–10 years, and more than 10 years. For inter-battery factor analysis (IBFA), multi-country and GARCH common factor analysis (MCFA and GARCH CFA), F_1 denotes the common factor and F_2 the country-specific factor. For principal component analysis (PCA), F_1 and F_2 are common factors and their loadings are given by the first two eigenvectors of the overall covariance matrix of bond returns.

With three countries, we can rewrite Eq. (2) as:

$$\Sigma_t = BB' + \begin{bmatrix} C_1C_1' + \Omega_{11t} & 0 & 0 \\ 0 & C_2C_2' + \Omega_{22t} & 0 \\ 0 & 0 & C_3C_3' + \Omega_{33t} \end{bmatrix}, \tag{17}$$

where B is a $3M \times 1$ matrix that contains the loadings on the common factor and C_k is a $M \times 1$ matrix that contains the loadings on the country k-specific factor and Ω_{kt} is:

$$\begin{bmatrix} \omega_{k1} + \alpha_{k1}\epsilon_{k1t}^2 + \beta_{k1}h_{k1t} & & 0 \\ & \ddots & \\ 0 & & \omega_{kM} + \alpha_{k1}\epsilon_{kMt}^2 + \beta_{k1}h_{kMt} \end{bmatrix} \tag{18}$$

for $k = 1, 2, 3$.[11] We model the residual variance of each bond return as a GARCH process where h_{km} is the conditional residual variance of bond return m, $m = 1, \ldots, M$, and $k = 1, 2, 3$. The residual covariance matrix is block-diagonal but, unlike in IBFA, it is time-varying. We can rewrite Eq. (17) as:

$$\Sigma_t = DD' + \Omega_t, \tag{19}$$

where D is a $3M \times 4$ matrix that contains the loadings on the common and country-specific factors and Ω_t is a diagonal matrix:

$$\Omega_t = \begin{bmatrix} \Omega_{11t} & 0 & 0 \\ 0 & \Omega_{22t} & 0 \\ 0 & 0 & \Omega_{33t} \end{bmatrix}. \tag{20}$$

The model parameters are estimated using maximum likelihood as in the previous model but allowing for a time-varying covariance matrix Σ_t (see Appendix 2 for details). When estimating the model, we assume that $\alpha_{km} = \alpha$ and $\beta_{km} = \beta$ in order to reduce the number of parameters.

This model is both more general and more restrictive than aforementioned models. In all other models, the covariance between bonds in different countries is driven exclusively by the common factor. Furthermore, the covariance between two bonds in the same country is determined by the loadings on the common factor and the residual country-specific covariances. While in both IBFA and multi-country common factor analysis, the country-specific factor structure is left unmodeled, it needs to be explicitly modeled when we incorporate heteroscedasticity. As a result, the loadings on the common factor will be influenced by the form of the country-specific residual volatility while in IBFA and multi-country common factor analysis, the common and local factor structures are extracted sequentially.

The loadings on the common and country-specific factors are displayed in Table 7 (see column headed GARCH CFA). The general pattern exhibited by the loadings in GARCH common factor analysis is consistent with the one exhibited by the loadings in IBFA and multi-country common factor analysis. There are some interesting differences though. The loadings in GARCH common factor analysis for US bonds are higher than in the other two common-local factor models. Also, the common factor loadings of German and Japanese bonds are lower in

[11] This model can easily be extended to more than one common and one country-specific factors.

GARCH common factor analysis. It is apparent that accounting for residual heteroscedasticity strengthens the US dominance of the common factor structure of international bond returns.

3.6. International price of risk

We estimate the price of risk of the common factor on US, German, and Japanese bond markets. The arbitrage pricing theory predicts that the expected returns are given by:

$$E(R) = B\lambda, \tag{21}$$

where B contains the loadings on the common factor and λ is the price of risk. It is common in asset pricing to estimate the price of risk using a regression of sample mean returns on the loadings. Cochrane (2001, p. 239) shows how the standard errors from this regression can be adjusted to account for the fact that the loadings have been estimated. However, this correction is derived under the assumption that the loadings were initially estimated by regressing asset returns on the factor returns. Such a correction is not appropriate here because we are extracting the factor loadings from the covariance matrix of returns using maximum likelihood estimation. Our procedure, which is based on the multi-country common factor analysis, solves this problem by jointly estimating the factor loadings and the price of risk.

To test the adequacy of the linear factor pricing model we introduce an intercept for each bond:

$$E(R) = \alpha + B\lambda \tag{22}$$

and test the null hypothesis that H_0: $\alpha = 0$ using a Lagrange multiplier test. When the factor loadings are estimated using multi-country common factor analysis, the estimate of the price of risk is 0.0680, with a standard error of 0.0541. The value of the Lagrange multiplier test is equal to 29.3849, which corresponds to a p-value of 0.0143. This result provides weak evidence against the linear global factor pricing model. Therefore, we cannot reject at the 1% confidence level that the US, German, and Japanese bond markets are integrated over our sample period.

To investigate the possibility of a time-varying price of risk, we estimate a model that treats the risk premium as a latent $AR(1)$ process:

$$\lambda_t = \lambda_0 + \delta_t, \tag{23}$$

where $\delta_t = \phi\delta_{t-1} + \eta_t$. We estimate this time-varying price of risk model using the Kalman filter. There is no evidence that this extended model is superior to a model with a constant price of risk. Even though this model has two extra parameters (ϕ and σ_η^2), the log-likelihood improves by less than one and the point estimate of σ_η^2 is less than one standard error from zero.

3.7. Application: hedging long-term bonds

We illustrate the economic usefulness of the different factor extraction techniques with a simple hedging example. Consider hedging a long-term bond using a portfolio of shorter-term bonds. This is done using alternatively domestic and foreign bonds. The hedged portfolio has to satisfy the following constraints.

$$\omega_M = 1,$$

$$\omega_1 + \omega_2 + \omega_3 + \omega_M = 0,$$

$$\omega_1 b_{1,1} + \omega_2 b_{1,2} + \omega_3 b_{1,3} + \omega_M b_{1,M} = 0,$$

$$\omega_1 b_{2,1} + \omega_2 b_{2,2} + \omega_3 b_{2,3} + \omega_M b_{2,M} = 0,$$

where ω_m is the fraction of the hedged portfolio invested in maturity-m bond, $b_{1,m}$ is the loading on the first factor for maturity-m bond, and $b_{2,m}$ is the loading on the second factor for maturity-m bond. The first equation reflects that we are hedging a long position in the long-term bond, the second equation constrains the strategy to be self-financing, while the last two equations state that the portfolio is immunized against the first two sources of risk. To hedge one bond against two sources of risk, we need a total of four bonds: the 10-year bond that is hedged and three other bonds with maturities in 1–3 years, 3–5 years, and 5–7 years, respectively.

Using matrix notations, we get:

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ b_{1,1} & b_{1,2} & b_{1,3} & b_{1,M} \\ b_{2,1} & b_{2,2} & b_{2,3} & b_{2,M} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_M \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (24)$$

$$XW = Y$$
$$W^* = X^{-1}Y.$$

The W^* matrix contains the weights of the self-financing hedged portfolio. We estimate the loadings on the risk factors using successively domestic and multi-country PCA, IBFA, multi-country and GARCH common factor analysis. For each method, we build a hedged portfolio and we report its volatility computed over our sample period.[12]

Table 8 presents the variances of the hedged portfolio returns for various target bonds and for, alternatively, domestic and foreign hedging strategies. The key point to take from this table is that hedging strategies developed using common and local factors dominate purely domestic and international PCA. Interestingly, when hedging long-term bonds using domestic bonds, strategies based on IBFA or multi-country common factor analysis dominate the other strategies. When hedging a long-term bond using foreign bonds, the best hedging performance is observed with either IBFA, multi-country or GARCH common factor analysis. PCA turns out to be systematically dominated, although this method allows hedging a long-term bond against two sources of risk with three other bonds vs. one source of risk with two other bonds with alternative methods. Accounting for country-specific factors improves the performance of domestic and international hedging strategies.

The models that account for both common and local factors also dominate domestic PCA. As reported in Table 2, the three shorter-term bonds in all three countries load positively on the first factor and negatively on the second factor, while the long-term bond loads positively on

[12] When hedging a long-term bond using foreign bonds and a common-local factor model, the hedging strategy has to be amended. Indeed, by construction, a foreign bond does not load on other country's domestic factors. As a result, for IBFA, multi-country and GARCH common factor analysis, we only hedge the long-term bond against one common factor using the shortest two foreign bonds.

Table 8
Hedging long-term bonds

	Domestic PCA	PCA	IBFA	MCFA	GARCH CFA
Panel A: Long-term US bond					
US	0.511	0.440	0.351	0.350	0.367
Germany	–	16.556	1.654*	1.438*	3.615*
Japan	–	23.303	2.422*	2.968*	6.580*
Panel B: Long-term German bond					
US	–	5.246	1.205*	1.286*	1.067*
Germany	12.132	3.500	0.719	0.936	1.569
Japan	–	15.931	–	2.752*	2.719*
Panel C: Long-term Japanese bond					
US	–	19.706	1.166*	1.103*	1.082*
Germany	–	32.482	–	1.090*	1.094*
Japan	1.260	0.568	0.747	0.531	0.623

Note: We hedge a long-term bond (more than 10-years) in country k using shorter-term bonds from country k or country l (1–3 years, 3–5 years, 5–7 years). For each factor extraction method, i.e., domestic principal component analysis (Domestic PCA), principal component analysis (PCA), inter-battery factor analysis (IBFA), multi-country and GARCH common factor analysis (MCFA and GARCH CFA), we present the volatility of the hedged portfolio computed over our sample period. *Denotes a hedging strategy where we only hedge a long-term bond against the common factor using the shortest two foreign bonds.

both factors, leading to extreme positions in the shorter-term bonds. This is a particularly severe problem for Germany since the loadings on the second factor for the three shorter-term bonds are virtually identical, leading to a near singular factor-loading matrix that produces such a poor performance.

4. Conclusion

In this paper, we analyze the common factor structure of Government bond returns of different maturities in several countries by formally accounting for the presence of country-specific factors. Unlike previous studies that use principal component analysis, we conclude that US bond returns share only one common factor with German and Japanese bond returns. Our result justifies the use of only one common factor in affine models in a multi-country setting. We find that the single common factor is associated most notably with changes in the level of the domestic term structures.

We also contribute to the literature on the extraction of common factors by extending the standard methodology to the case of more than two countries and by allowing for time-variation in the covariance matrix of bond returns. These extensions preserve the intuition of the inter-battery factor analysis of Tucker (1958) since they allow for the presence of common and country-specific factors. We show that accounting for country-specific factors improves the performance of domestic and international hedging strategies.

Future research may benefit from the framework presented in the present paper. It may be used to estimate common factor structures in presence of several countries or currency areas. Moreover, it may also be suitable to study comovements between bond returns from different credit classes, hedge-fund performances grouped by fund types, measures of asset liquidity in different market segments, or implied volatilities arranged by option maturities.

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Appendix 1. Common principal component analysis

The problem of estimating common principal components in K groups can be translated to the problem of diagonalizing jointly several positive-definite covariance matrices:

$$\Sigma_k = A\Lambda_k A', \quad k = 1, \ldots, K.$$

Following Flury (1988), estimates of $\widehat{A}$ and $\widehat{\Lambda}_k$ can be obtained taking advantage of the Hadamard inequality, which states that for any positive-definite symmetric matrix H, one has $\det H \leq \det \operatorname{diag}(H)$, with equality if and only if H is diagonal. Diag(H) means the diagonal matrix with the same diagonal elements as H. Consequently $\det \operatorname{diag}(H)$ is the product of all diagonal elements of H. A can be estimated by minimizing the following function:

$$\Phi(A) = \prod_{k=1}^{K} \left[\frac{\det \operatorname{diag}(A'\Sigma_k A)}{\det(A'\Sigma_k A)} \right].$$

Minimizing $\Phi(A)$ can be viewed as trying to find a matrix A which diagonalizes jointly the K covariance matrices Σ_k as much as it can. This result means that the common principal component transformation can be viewed as a rotation yielding variables that are as uncorrelated as possible simultaneously in K groups. Moreover, Flury (1988) has proved that the A matrix which minimizes Φ(A) is the maximum likelihood estimate of A in the common principal component model under the assumption that Σ_k follows a Wishart distribution. The FG-algorithm executes this numerical minimization and Fortran routines have been provided by Flury (1988, Appendix C). A detailed description and proof of convergence can be found in Flury (1988, p. 178). Robust common principal components are derived in Boente et al. (2002).

Appendix 2. GARCH common factor analysis

This appendix details the estimation of the factor loadings in the GARCH common factor analysis presented in Section 3.5. Denote by ξ_t the vector of all factor returns in period t: $\xi_t = [F_t, F_{1t}, F_{2t}, F_{3t}]$ where F_t is the return on the common factor and F_{kt} is the return on the local factor for country k. We posit a GARCH process for the residual volatility from the factor model:

$$R_t = E(R) + D\xi_t + \epsilon_t,$$

where the latent factors are i.i.d. multivariate normal and uncorrelated with the residuals:

$$\xi_t \sim \text{MVN}(0, I), \quad E\left(\xi_t \xi'_{t-i}\right) = \begin{matrix} 0, i \neq 0 \\ I, i = 0, \end{matrix} \quad \text{and } E\left(\xi_t \epsilon'_{t-1}\right) = 0.$$

Inference about D can be derived by noting that $R_t | I_{t-1} \sim \text{MVN}(\mu, \Sigma)$, where $\mu = E_{t-1}(R_t) = E(R)$ is the mean vector and the covariance matrix is given by

$\Sigma = E_{t-1}[(R_t - \mu)(R_t - \mu)'] = DD' + \Omega$. Because R_t and ξ_t are multivariate normal, inference about the latent state variable is easily obtained:

$$\widehat{\xi}_{t|t} = D'(DD' + \Omega)^{-1}(R_t - \mu), \quad \widehat{P}_{t|t} = I - D'(DD' + \Omega)^{-1}D,$$

where $\widehat{\xi}_{t|t} = E(\xi_t|R_t)$ is the conditional expected value of ξ_t given the asset returns R_t and $\widehat{P}_{t|t} = E[(\widehat{\xi}_{t|t} - \xi_t)(\widehat{\xi}_{t|t} - \xi_t)]$, its associated mean-squared-error. An interesting limiting case is when D has full column rank and $\Omega = 0$, such as in principal component analysis, in which case $\widehat{\xi}_{t|t} = D'(R_t - \mu)$ and mean-squared-error 0.

We incorporate heteroscedasticity by modeling the conditional residual volatility of bond returns in country k as a GARCH process:

$$h_{kt} = \omega_k + \alpha_k \epsilon_{kt-1}^2 + \beta_k h_{kt-1}.$$

Unfortunately, because the factor returns are latent we do not observe the residuals and we therefore use the unobserved components GARCH model of Harvey et al. (1992). We define the conditional volatility as $h_{kt} = E_{t-1}(\epsilon_{kt}^2)$ and replace the unobserved lagged squared residual with its expectation. We decompose the unobserved true residual into its expected and unexpected components $\epsilon_{kt-1} = \widehat{\epsilon}_{kt-1} + (\epsilon_{kt-1} - \widehat{\epsilon}_{kt-1})$, noting that $\epsilon_{kt-1} - \widehat{\epsilon}_{kt-1} = D_k (\xi_{t-1} - \widehat{\xi}_{t-1|t-1})$, giving:

$$E_{t-1}\left(\epsilon_{kt-1}^2\right) = \widehat{\epsilon}_{kt-1}^2 + D_k \widehat{P}_{t-1|t-1} D_k',$$

where D_k is the kth column of D. Conditional volatility is then given by the recursion:

$$h_{kt|t-1} = \omega_k + \alpha_k \left(\widehat{\epsilon}_{kt-1}^2 + D_k \widehat{P}_{t-1|t-1} D_k' \right) + \beta_k h_{kt-1|t-2}.$$

Note that we use $h_{kt-1|t-2}$ rather than $h_{kt-1|t-1}$ for lagged conditional variance which preserves the recursive nature of volatility and dramatically simplifies model estimation. We initialize each volatility process using the unconditional volatility $\sigma_k^2 = E(h_k) = \omega_k/(1 - \alpha_k - \beta_k)$. The log-likelihood is constructed as $\mathrm{LL}(\theta; R) = \sum_{t=1}^{T} \log f(R_t; \mu, \widehat{\Sigma}_{t|t-1})$ where θ is the parameter vector, $\hat{\Sigma}_{t|t-1} = DD' + \widehat{\Omega}_{t|t-1}$ is the conditional covariance matrix of R_t, and $\widehat{\Omega}_{t|t-1}$ is the conditional covariance matrix of the residuals. The updating equations for the latent state variables are given by:

$$\widehat{\xi}_{t|t} = D'\left(DD' + \Omega_{t|t-1}\right)^{-1}(R_t - \mu), \quad \widehat{P}_{t|t} = I - D'\left(DD' + \Omega_{t|t-1}\right)^{-1}D.$$

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