b) $y' + \frac{2}{t} y = \frac{\sin t}{t}$ $\quad t \neq 0$; $y(\pi/2) = 1$

~~This is not an initial value problem~~

Using integrating factor again,

$$\mu' = \frac{2}{t}\mu \Rightarrow \ln \mu = 2 \ln t + \cancel{C} \quad \text{(set = 0)}$$

$$= \ln t^2 \quad , \quad (t > 0)$$

$$\Rightarrow \boxed{\mu = t^2}$$

(because I want $\frac{\pi}{2}$ to be in)

Then, $(\mu y)' = \frac{\sin t}{\cancel{t}} t^{\cancel{2}}$.

Integrate,

$$\mu y = \int t \sin t \, dt + C \ ,$$

Then $$y = \frac{-t\cos t + \sin t + C}{t^2}$$

Using the initial condition,

$$1 = \frac{1 + C}{(\pi/2)^2} \rightarrow C = \left(\frac{\pi}{2}\right)^2 - 1$$

$$\therefore \quad y = \frac{-t\cos t + \sin t + \left(\frac{\pi}{2}\right)^2 - 1}{t^2}$$

$$Dom(y) = (0, +\infty)$$

■