On Quiz #1 (make-up)

- Solve the initial value problems and state the domain of the solution:

a) $t\,y' + (t+1)\,y = t \quad ; \quad y(\ln 2) = 1$

$t \neq 0 \rightsquigarrow \quad y' + \left(\frac{t+1}{t}\right) y = 1 \quad \ldots$ (linear)

Using integrating factor method:

$$\mu y' + \mu\left(\frac{t+1}{t}\right) y = \mu \quad \cdots \ (*)$$

Want $\quad \mu' \cancel{y} = \mu\left(\frac{t+1}{t}\right) \cancel{y}$

then $\quad \frac{\mu'}{\mu} = \frac{t+1}{t}$

Integrating with respect to t.

$$\ln \mu = t + \ln t + C \quad , \ t > 0$$

pick $C = 0$ and exponentiate both sides to get

$$\mu = e^{t+\ln t} = e^t \cdot e^{\ln t} = e^t \cdot t$$

Back to $*$, we have

$$\underbrace{\mu y' + \mu' y}_{} = \mu$$

$$(\mu y)' = \mu$$

integrate,

$$\mu y = \int t e^t dt + K \quad , K \text{ ct.}$$

$$\boxed{y = \frac{te^t - e^t + K}{te^t}}$$

Using the initial condition,

$$1 = y(\ln 2) = 1 - \frac{1}{\ln 2} + \frac{C}{2\ln 2} \Rightarrow \boxed{C = 2}$$

$\therefore \ y(t) = 1 - \frac{1}{t} + \frac{2}{te^t} \quad ; \ \mathrm{Dom}(y) = (0, +\infty)$ ∎