

## § 2.3 Solutions to some homework <sup>and quiz</sup> problems

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- 7)  $\left. \begin{array}{l} K: \text{deposit per year} \\ r: \text{annual rate of return} \\ t: \text{(measured in years)} \end{array} \right\} S(t): \text{investment at } t.$

Model:  $\left. \begin{array}{l} \frac{dS}{dt} = rS + K \\ S(0) = 0 \end{array} \right\} \text{I.V.P.}$

a) Solving I.V.P.

$$\int \frac{dS}{rS+K} = \int dt \leadsto \ln(rS+K) = rt + C$$

$$rS + K = Ce^{rt} \leadsto S(t) = Ce^{rt} - \frac{K}{r}$$

$$0 = S(0) = C - \frac{K}{r} \leadsto \boxed{S(t) = \frac{K}{r} (e^{rt} - 1)}$$

b)  $r = 7.5\% = 0.075$

$K = ?$  so that  $S(40) = 1,000,000$

$$\frac{K}{0.075} (e^{0.075(40)} - 1)$$

$$\Rightarrow \frac{10^3 \times 75}{e^3 - 1} = K \leadsto \boxed{K \cong 3,930 \text{ dollars year.}}$$