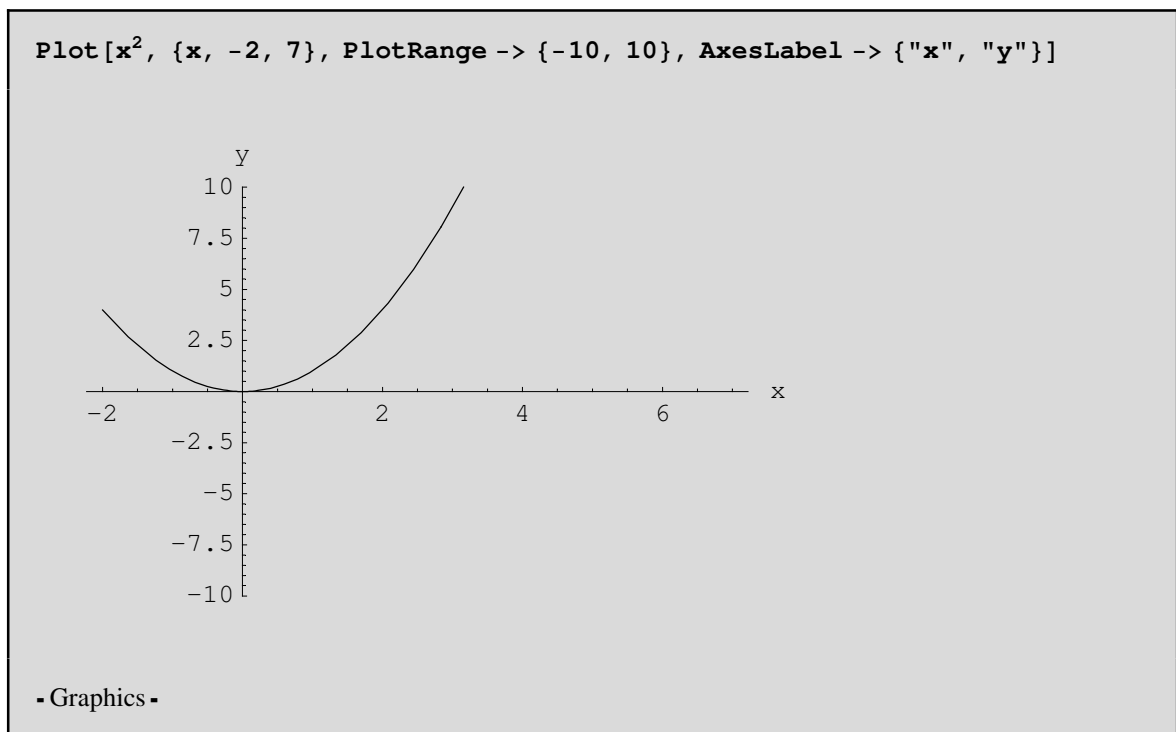*Ruben Agapito Ruiz*
Math 24
Summer Session II, 2003

Ordinary Differential Equations

Problem 2

We are gonna learn some *Mathematica* commands: (*Mathematica* commands always start with a capital letter. In other words, *Mathematica* is case sensitive.)

```
Plot[x², {x, -2, 7}, PlotRange -> {-10, 10}, AxesLabel -> {"x", "y"}]
```


- Graphics -

How to get more information about a *Mathematica* command:

```
? Plot
```

```
Plot[f, {x, xmin, xmax}] generates a plot of f as a
    function of x from xmin to xmax. Plot[{f1, f2, ... },
    {x, xmin, xmax}] plots several functions fi. More...
```

but now how to get even more information about a command:

```
?? Plot
```

Plot[f, {x, xmin, xmax}] generates a plot of f as a
    function of x from xmin to xmax. Plot[{f1, f2, ... },
   {x, xmin, xmax}] plots several functions fi. More...

Attributes[Plot] = {HoldAll, Protected}

Options[Plot] =
 {AspectRatio → $\frac{1}{GoldenRatio}$, Axes → Automatic, AxesLabel → None,
  AxesOrigin → Automatic, AxesStyle → Automatic, Background → Automatic,
  ColorOutput → Automatic, Compiled → True, DefaultColor → Automatic,
  Epilog → {}, Frame → False, FrameLabel → None, FrameStyle → Automatic,
  FrameTicks → Automatic, GridLines → None, ImageSize → Automatic,
  MaxBend → 10., PlotDivision → 30., PlotLabel → None, PlotPoints → 25,
  PlotRange → Automatic, PlotRegion → Automatic, PlotStyle → Automatic,
  Prolog → {}, RotateLabel → True, Ticks → Automatic,
  DefaultFont ⧴ $DefaultFont, DisplayFunction ⧴ $DisplayFunction,
  FormatType ⧴ $FormatType, TextStyle ⧴ $TextStyle}

```
Plot3D[Sin[x y], {x, -5, 5}, {y, -5, 5}, PlotPoints -> 40]
```


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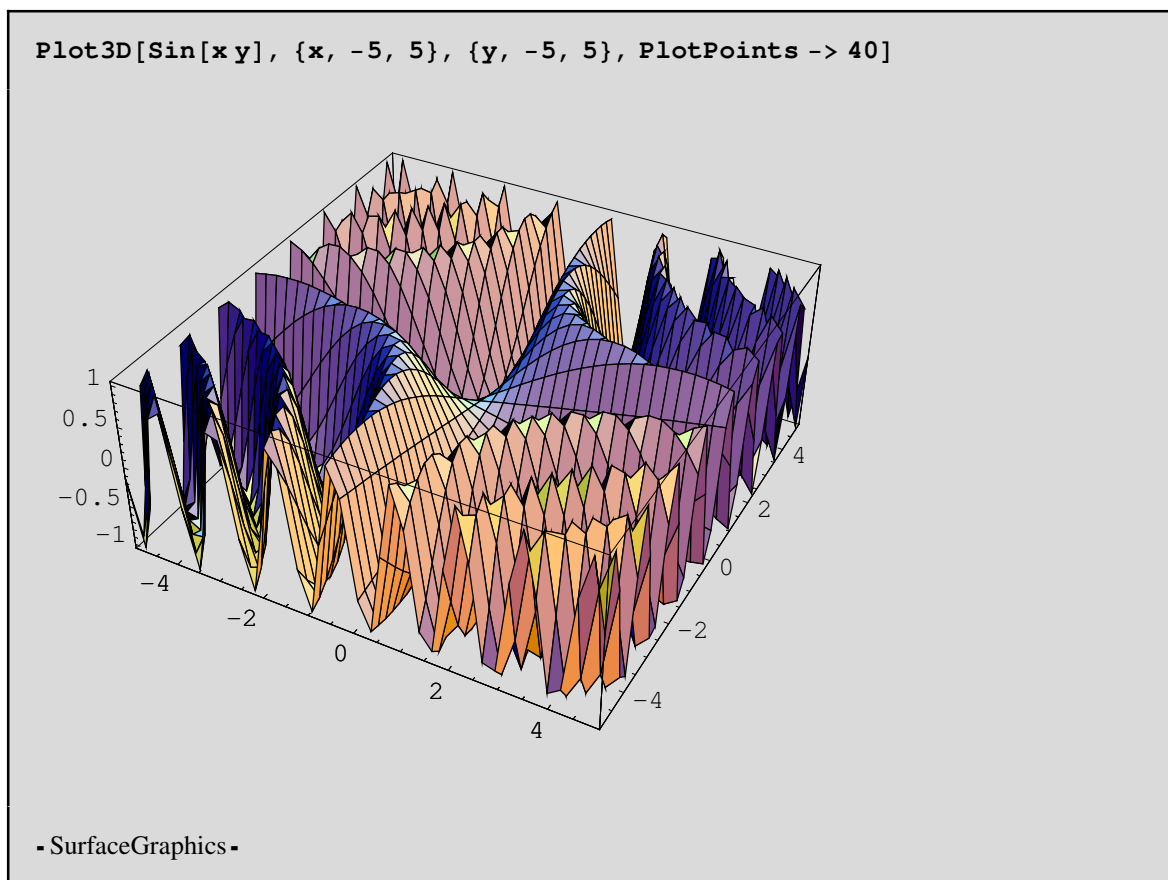
ODE commands

■ Example 1

To solve the first-order differential equation $\frac{dy}{dx} = x + y$, we simply type

```
DSolve[y'[x] == x + y[x], y[x], x]
```

$$\{\{y(x) \rightarrow -x + e^x c_1 - 1\}\}$$

Let's get more information about DSolve

```
?? DSolve
```

```
DSolve[eqn, y, x] solves a differential equation
    for the function y, with independent variable x. DSolve[
    {eqn1, eqn2, ...  }, {y1, y2, ...  }, x] solves a
    list of differential equations. DSolve[eqn, y, {x1,
    x2, ... }] solves a partial differential equation. More...
```

```
Attributes[DSolve] = {Protected}

Options[DSolve] = {DSolveConstants → C}
```

■ Example 2

```
g = #1^2 #2 + 3 #1 &;   (* this is a pure function, this is a comment  *)
g[3, 4]
```

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To obtain the solution of $\frac{dy}{dx} = x + y$ as a pure function, we enter

```
solution = DSolve[y'[x] == x + y[x], y, x]
```

$\{\{y \rightarrow \text{Function}[\{x\}, -x + e^x c_1 - 1]\}\}$

If we want to evaluate the solution , we can type

```
y[x] /. solution (* the symbol /. means evaluation  *)
```

$\{-x + e^x c_1 - 1\}$

Using pure functions you can evaluate derivatives of the solution:

```
y'[x] /. solution
```
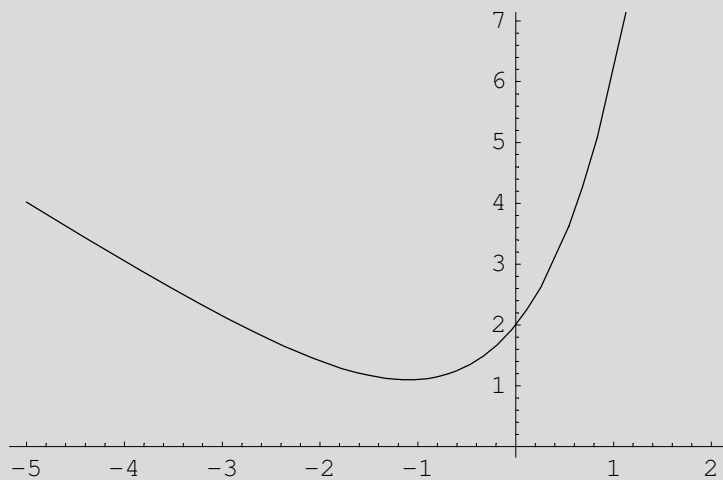
$\{e^x c_1 - 1\}$

■ Example 3. ODE with initial conditions

Solve the ODE $\frac{dy}{dx} = x + y$ with the initial condition $y(0) = 2$. Then plot the solution

```
equation = DSolve[{y'[x] == x + y[x], y[0] == 2}, y[x], x]
```

$\{\{y(x) \rightarrow -x + 3\,e^x - 1\}\}$
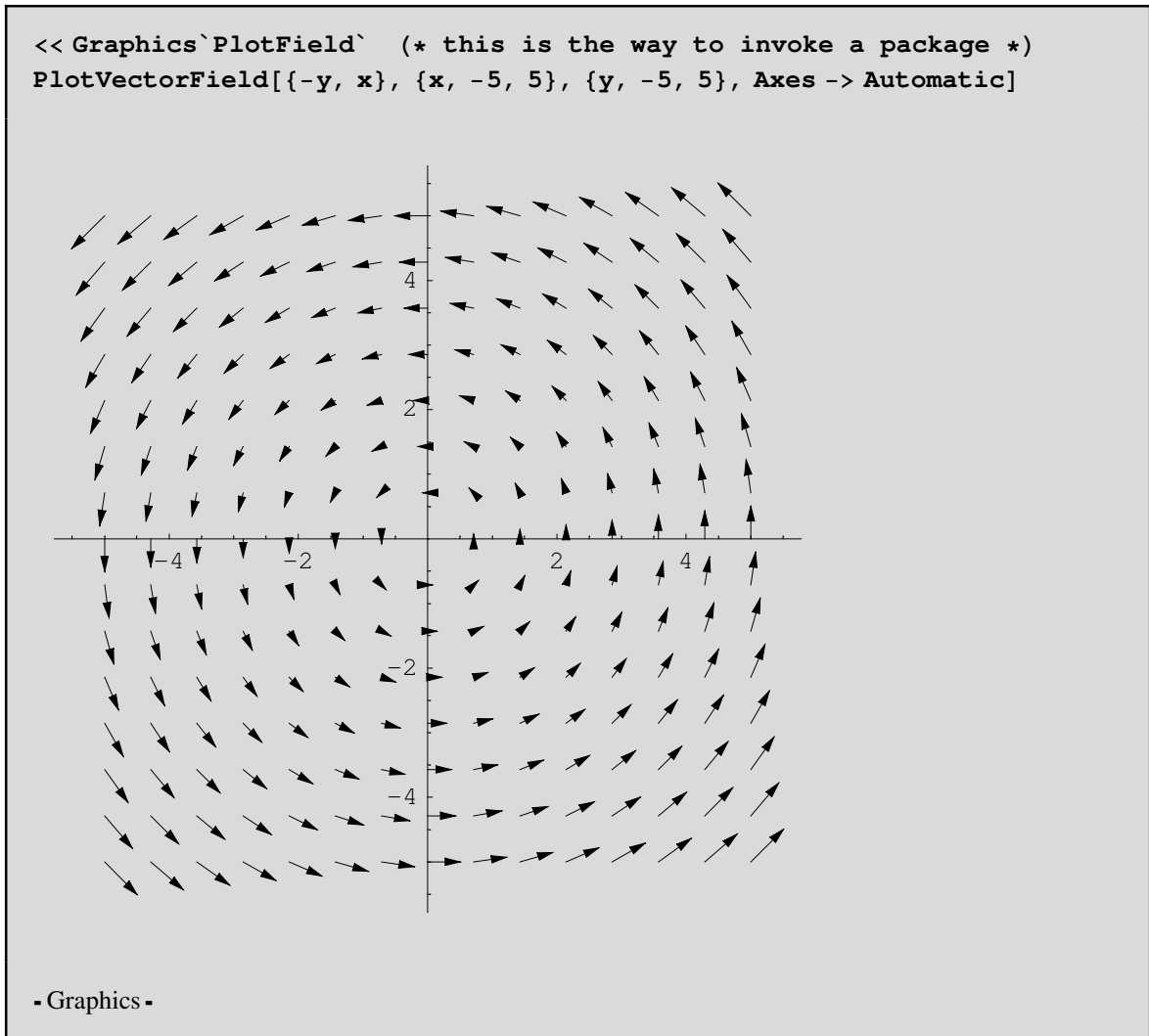
```
Plot[y[x] /. equation, {x, -5, 2}]
```


- Graphics -

■ Example 4. How to plot Vector Fields

Plot the vector field $F(x, y) = -y\,i + x\,j$. By default, no axes are drawn so the option Axes->Automatic will be used.

```
<< Graphics`PlotField`  (* this is the way to invoke a package *)
PlotVectorField[{-y, x}, {x, -5, 5}, {y, -5, 5}, Axes -> Automatic]
```


- Graphics -

■ some example

a common mistake in *Mathematica*. Use commands without invoking packages. For example

```
ImplicitPlot[x² + y² == 1, {x, -1, 1}]
```

ImplicitPlot$(x^2 + y^2 == 1, \{x, -1, 1\})$
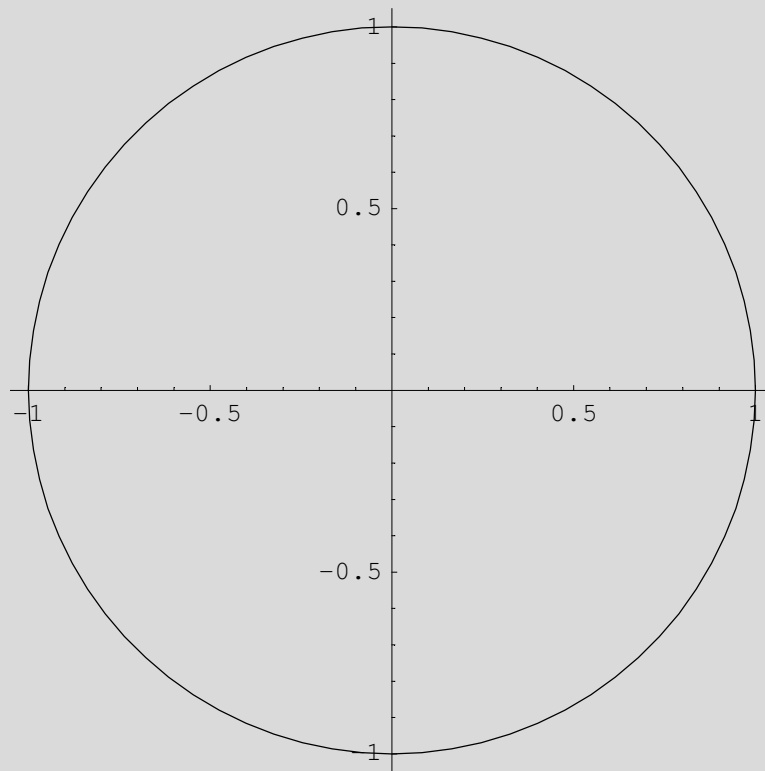
So we invoke the package:

```
<< Graphics`ImplicitPlot`
```

— *ImplicitPlot::shdw : Symbol ImplicitPlot appears in multiple contexts {Graphics`ImplicitPlot`, Global`};*
definitions in context Graphics`ImplicitPlot` may shadow or be shadowed by other definitions.

So in order to graph the equation we need to remove the variable we created

```
Remove[ImplicitPlot]
```

```
ImplicitPlot[x² + y² == 1, {x, -1, 1}]
```


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