Quiz # 2

1. Evaluate the integral $\displaystyle\iint_{\mathcal{R}} y\,dA$, where $\mathcal{R}$ is the bounded region in the first quadrant delimited by the circle $x^2+y^2=9$ and the lines $y=x$ and $y=0$.

   Solution. It is convenient to use polar coordinates. Thus, we have

$$\iint_{\mathcal{R}} y\,dA = \int_0^{\frac{\pi}{4}}\int_0^3 r^2\sin\theta\,dr\,d\theta = \left(\int_0^3 r^2\,dr\right)\left(\int_0^{\frac{\pi}{4}}\sin\theta\right) = 9\left(\frac{2-\sqrt{2}}{2}\right).$$

2. Evaluate $\displaystyle\iiint_{\mathcal{E}} (x^2+y^2+z^2)\,dV$, where $\mathcal{E}$ is the unit ball $x^2+y^2+z^2\le 1$.

   Solution. It is convenient to use spherical coordinates. Thus, we have

$$\iiint_{\mathcal{E}} (x^2+y^2+z^2)\,dV = \int_0^{2\pi}\int_0^{\pi}\int_0^1 \rho^4\sin\varphi\;d\rho\,d\varphi\,d\theta = \frac{4\pi}{5}.$$