

Quiz # 1

1. Evaluate the path integral of $\exp \sqrt{z}$ along $\vec{\alpha}(t) = (1, 2, t^2)$, where $0 \leq t \leq 1$.

Solution.

$$\begin{aligned} \int_{\Gamma} \exp \sqrt{z} \, ds &= \int_0^1 (\exp \sqrt{t^2}) \sqrt{(2t)^2} \, dt \\ &= \int_0^1 2te^t \, dt = 2 \left(te^t \Big|_0^1 - \int_0^1 e^t \, dt \right) \\ &= 2(e - (e - 1)) = 2. \end{aligned}$$

□

2. Evaluate $\int_{\Gamma} x^2 dx - xydy + dz$, where Γ is the parabola $z = x^2$, $y = 0$ from $(-1, 0, 1)$ to $(1, 0, 1)$.

Solution. This parabola may be parameterized by $x = t$, $z = t^2$, $y = 0$, where $-1 \leq t \leq 1$. Then,

$$\begin{aligned} \int_{\Gamma} x^2 dx - xydy + dz &= \int_{-1}^1 (t^2 dt - 0dt + 2t dt) \\ &= \left(\frac{t^3}{3} + 2\frac{t^2}{2} \right) \Big|_{-1}^1 \\ &= \frac{1}{3} + 1 + \frac{1}{3} - 1 = \frac{2}{3}. \end{aligned}$$

□