Homework # 2

1. (§5.1 (1c)) Evaluate $\int_0^1 \int_0^1 (xye^{x+y})\, dy\, dx$.

2. (§5.2 (6)) Compute the volume of the solid bounded by the surface $z = \sin y$, the planes $x = 1$, $x = 0$, $y = 0$ and $y = \frac{\pi}{2}$, and the XY plane.

3. (§5.3 (2a)) Evaluate and sketch the region of integration, $\int_{-3}^{2} \int_0^{y^2} (x^2 + y)\, dx\, dy$.

4. (§5.4 (2a)) Find $\int_{-1}^{1} \int_{|y|}^{1} (x + y)^2\, dx\, dy$.

5. Prove that

   (a) $\int_0^1 \left\{ \int_0^1 \frac{x - y}{(x + y)^3}\, dy \right\} dx = \frac{1}{2}$.

   (b) $\int_0^1 \left\{ \int_0^1 \frac{x - y}{(x + y)^3}\, dx \right\} dy = -\frac{1}{2}$.

   What happened with Fubini's theorem?

6. Compute $\int_0^a \int_0^z \int_0^y e^{(a-x)^3}\, dx\, dy\, dz \quad (a > 0)$.

7. * Prove that

$$\int_0^x \left\{ \int_0^y \left[\int_0^t f(u)\, du \right] dt \right\} dy = \frac{1}{2} \int_0^x (x - u)^2 f(u)\, du.$$

   Hint. Set $I(x) = \int_0^x \left\{ \int_0^y \left[\int_0^t f(u)\, du \right] dt \right\} dy$ and $J(x) = \frac{1}{2} \int_0^x (x - u)^2 f(u)\, du$, and then take derivatives. We have not put conditions on f. Assume what you need to work out this problem.

8. Calculate $\iint_{\mathcal{R}} \frac{1}{x + y}\, dy\, dx$, where $\mathcal{R}$ is the region bounded by $x = 0$, $y = 0$, $x + y = 1$ and $x + y = 4$, by using the mapping $T(u, v) = (u - uv, uv)$.

9. Integrate $x^2 + y^2 + z^2$ over the cylinder $x^2 + y^2 \leq 2$, $-2 \leq z \leq 3$.

10. Find the center of mass of the region between $y = 0$ and $y = x^2$, where $0 \leq x \leq \frac{1}{2}$.