SOLUTIONS TO THE FINAL REVIEW

1. $23x - 27y + 84z = 64$.
2. There is no real solution for a.
3. $-3/\sqrt{5}$.
4. (a) It doesn't exist, take for instance $y = x$ and let $x \to 0$, you get 1, on the other hand, if $x = 0$ and $y \to 0$, then the limit is 0.
   (b) The limit does exist and is $e^{-1}/2$.
5. (a) $df = -y\sin(xy+z)dx - x\sin(xy+z)dy - \sin(xy+z)dz$.
   (b) $df = [2e^{-3t}\cos(2x+5t)]dx + [-3e^{-3t}\sin(2x+5t) + 5e^{-3t}\cos(2x+5t)]dt$
   (c) $df = ydx + (1/y + x)dy$
   (d) $df = [\frac{\sec^2 x}{2\sqrt{\tan x + \arctan x}}]dx + [\frac{1}{2(1+y^2)\sqrt{\tan x + \arctan y}}]dy$
6. (a) $\nabla f(0,0) = (0,0)$ and $Hf(0,0) = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$
   (b) $\nabla f(0,-3) = (0,0)$ and $Hf(0,-3) = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}$
7. (a) Local minimum at $(-1/4, -1/4)$.
   (b) Saddle at $(0,0)$.
   (c) Saddle points at $(1,1)$ and $(-1,-1)$.
   (d) Local minimum at $(5, 27/2)$ and a saddle at $(1, 3/2)$.
8. The global maximum value is 2 and is attained at the points $(0, \pi/2)$, $(2\pi, \pi/2)$ and the global minimum value is -2, attained at the point $(\pi, 3\pi/2)$.
9. $(40/9, -20/9, 40/9)$.
10. The absolute maximum value is $3/2$ attained at the points $(\sqrt{2}/2, \sqrt{2}/2)$ and $(-\sqrt{2}/2, -\sqrt{2}/2)$ and the absolute minimum value is 0, attained at $(0,0)$.
11. (a) $5\pi/2$.
    (b) $-1/6$.
12. (a) 26.
    (b) $-1/(2\pi)$
13. 9.

14. In cartesian coordinates,

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} 9 - x^2 - y^2 \, dydx,$$

or in polar coordinates,

$$\int_{0}^{2\pi}\int_{0}^{3} (9 - r^2) r \, drd\theta.$$

15. 1/96.

16. $486\pi(\frac{\sqrt{2}-1}{5})$.

17. (a) $-66/125$.
    (b) $\frac{3}{2}\sin 1$.