MATH 22, REVIEW EXERCISES FOR THE FINAL

(1) Find the equation of the tangent plane to the surface determined by the equation $y^3x - xz^2 + z^5 = 9$ at the point $P(-1, 3, 2)$.

(2) If the point $p = (a^2, -2a, -1)$ lies on the tangent plane to the surface $z = e^x/y$ at the point $(0, 1, 1)$, what is the value of the constant a?

(3) Find the directional derivative of the function $f(x, y) = \ln(x^2 + y^2)$ at the point $(-1, 1)$ along the direction of the vector $\mathbf{v} = 2\mathbf{i} - \mathbf{j}$.

(4) Determine whether the following limits exist or not. If so, compute its value:

  (a) $\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$.

  (b) $\lim_{(x,y)\to(0,-1)} \frac{e^{x^2-y^2}}{x^2+y^2+1}$

(5) Find the differential of the functions in Problems (a)-(d).

  (a) $f(x, y, z) = \cos(xy + z)$

  (b) $f(x, t) = e^{-3t} \sin(2x + 5t)$

  (c) $f(x, y) = \ln(ye^{xy})$

  (d) $f(x, y) = \sqrt{\tan x + \arctan y}$

  Hint: $\frac{d}{dx} \tan x = \sec^2 x$ and $\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$. Do not forget to use the chain rule.

(6) Find the gradient ∇f and the hessian Hf at the given point.

  (a) $f(x, y) = (x - y)^2$; $(0, 0)$.

  (b) $f(x, y) = e^{(y+3)^2} \cos x$; $(0, -3)$.

(7) Find the critical points of the functions (a)-(e), and classify them.

  (a) $f(x, y) = 3y^2 + 2xy + 2y + x^2 + x + 4$

  (b) $f(x, y) = e^{1+x^2-y^2}$

  (c) $f(x, y) = (x - y)(xy - 1)$

  (d) $f(x, y) = x^3 + y^2 - 6xy + 6x + 3y$.

  (e) $f(x, y, z) = x^3 - y^3 - 2xy + z^2$.

(8) Find the global maximum and minimum values for $f(x, y) = \cos x + \sin y$ on the rectangle $R = [0, 2\pi] \times [0, 2\pi]$.

(9) Find the point in the plane $2x - y + 2z = 20$ nearest the origin.

(10) Consider the function $f(x, y) = x^2 + xy + y^2$ on the region

$$R = \{(x, y)|x^2 + y^2 \leq 1\}.$$

Use the method of Lagrange multipliers to find the maximum and minimum values for f on the unit circle (the unit circle is $\{(x, y)|x^2 + y^2 = 1\}$). Use this to determine the absolute(global) maximum and minimum values for f on R.

(11) Evaluate the integrals.

  (a) $\int_{-1}^{1} \int_0^{\sqrt{1-x^2}} 5\, dy\, dx$

  (b) $\int_{\pi/2}^{\pi} \int_0^{\sin y} x \cos y\; dx\, dy$;

(12) Evaluate the given integrals after sketching the region of integration.

  (a) $\int_0^6 \int_{x/3}^2 x\sqrt{y^3 + 1}\, dy\, dx$;

(b) $\int_0^3 \int_{y^2}^9 y\sin(\pi x^2)\,dx\,dy$;

(13) Find the volume of the region under the graph of $f(x,y) = e^x y^2$ and above the rectangle $0 \le x \le \ln 2$, $0 \le y \le 3$ in the xy-plane.

(14) Set up, but do not evaluate, the integral that computes the volume of the solid under the graph of $f(x,y) = 10 - x^2 - y^2$ and above the plane $z = 1$.

(15) Evaluate the integral by changing to cylindrical coordinates

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \int_{x^2+y^2}^{\sqrt{x^2+y^2}} xyz\,dzdxdy$$

(16) Evaluate the integral by changing to spherical coordinates

$$\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} (x^2+y^2+z^2)\,dzdxdy$$

(17) Evaluate the integral by making an appropriate change of variables.

(a) $\int\int_R xy\,dA$ where R is the region bounded by the lines $2x - y = 1$, $2x - y = -3$, $3x + y = 1$ and $3x + y = -2$.

(b) $\int\int_R \cos\frac{y-x}{y+x}\,dA$ where R is the trapezoidal region with vertices $(1,0), (2,0), (0,2), (0,1)$.