English summary

Remarkably just a few months after Einstein published his famous theory of general relativity, Schwarzschild discovered the first non-trivial spacetime solution satisfying the field equations. It was understood only a few decades later that this solution represented an asymptotically flat spacetime containing a black hole as an isolated system. Since their discovery, black holes have been a fascinating topic to both scientist and lay people and have therefore been subject of countless scientific investigations but also science fiction novels and movies. Thanks to the growing popularity of black holes it is almost common knowledge that from the exterior boundary of black holes, the so called event horizon not even light can escape and any object crossing it will be swallowed by the black hole. The interior region of black hole spacetimes is considered mysterious and while in 70's stories the astronaut always smashes in the black hole, in later material – not least due to the recent movie "Interstellar" – the rumor has spread that there might be a chance for survival.

Mathematically this can be understood as follows. There exist four known exact solutions to the Einstein-Field equations, which represent black hole spacetimes. The simplest of these is the Schwarzschild solution, which is a spherically symmetric, static solution without charge or angular momentum but the mass of the black hole as its only parameter. In Schwarzschild spacetimes it is indeed true that an observer entering the interior inevitably has to be destroyed by the growing tidal forces as he approaches the singularity of the black hole. The nature of the singularity in this spacetime is strong in the sense that a macroscopic object would suffer infinite deformation and also in the sense that the spacetime is not extendible at the singularity. In fact classically the singularity itself should not even be considered part of the spacetime since the theory of general relativity simply breaks down there. Opposed to this, curvature does not blow up in the interior of charged, spherically symmetric static black holes, so called Reissner-Nordström black holes. Instead this solution shows a different peculiar feature: The solution is only unique up to a boundary which is called the Cauchy horizon or sometimes the inner horizon. Beyond the Cauchy horizon determinism is lost while the spacetime remains regular. Roughly speaking, neglecting the backreaction of the observer's mass onto the background, this implies that our astronaut could travel out of the black hole into another universe causally disconnected from ours. His further fate cannot by any means be predicted by classical theory. The same characteristic shows in the Kerr solution which is an axialsymmetric, stationary solution representing rotating black holes and in the Kerr-Newmann solution which represents a charged rotating black hole solution. Being aware that general relativity is a dynamical theory and its exact solutions constitute only very specific cases we may now wonder if the regularity of interior spacetimes is a stable feature or if under small perturbations singularities might emerge. In particular, a specific type of singularities evolving under perturbations –namely strong ones such as occurring

in Schwarzschild spactime– would indicate that the doorway to the causally disconnected universe is closed. The result of this thesis does not advocate this possibility, which would exclude the appearance of the non-deterministic regions. Instead we obtain certain stability results under perturbations which however leave some space for irregular behavior at the Cauchy horizon without completely closing the exit. In the following we will specify further which exact setup leads to this conclusion.

First of all we have to decide which spacetime we want to perturb. Of the above mentioned black hole solutions the Kerr solution is the astrophysically most relevant solution. Astrophysical objects are not expected to be significantly charged since the induced electric field is expected to balance itself out by accreting opposed charges. Nevertheless, due to the similarity of the causal structure, the Reissner-Nordström solution is a very important candidate as a toy model for the mathematically much more complicated Kerr solution. Therefore, in this thesis we will focus on the Reissner-Nordström solution.

Second the simplest perturbations one could consider are linear perturbations. Obviously, in order to finally analyze non-linear perturbations it is crucial to have a very good understanding of its linearized version first. On the other hand since general relativity is a non-linear theory the reader might be skeptical in how far conclusions derived within a linearized model can indicate anything physically relevant. Fortunately, for reasons too involved to explain here[1] there is good evidence to believe that the stability mechanism is indeed already captured by the linearized theory. Analyzing linear perturbations in full Einstein-Field equations is still a complicated problem due to the tensorial structure of the equations. Therefore, as a toy model for linear perturbations we consider solutions to scalar wave equations on fixed Reissner-Nordström background. Given this setup we prove boundedness of the wave equation in the entire interior up to and including the Cauchy horizon. This result is a new mathematical insight in its own right. Going one step further and taking it serious as a model for the physical fate of perturbed Reissner-Nordström spacetime it hints to the fact that perturbation will not lead to a strong spacelike singularity. In fact it leaves space for at most a weak null singularity along the Cauchy horizon so that the travel of our astronaut to the causally disconnected and undetermined region is not a priori prevented.

[1]We will catch up with this in part I of the thesis.