Exercises

September 2, 2019

1. Suppose that X is an irreducible Markov chain. Suppose that there exists i in the state space such that $\mathbb{P}_i(T_i < \infty) = 1$. Show that for all j we also have $\mathbb{P}_j(T_j < \infty) = 1$.

2. Show that simple random walk on $\mathbb{Z}^d$ for all $d \geq 4$ is transient.

3. Show that a chain with matrix P and invariant distribution π is reversible if and only if for all x, y we have

$$\pi(x)P(x,y) = \pi(y)P(y,x).$$

4. (i) Let X be a simple symmetric random walk on $\mathbb{Z}_n$. Find its invariant distribution. Is the chain reversible?

(ii) Let X be a biased random walk on $\mathbb{Z}_n$ with transition probabilities $P(i, (i+1) \bmod n) = 2/3$ and $P(i, (i-1) \bmod n) = 1/3$. Find its invariant distribution. Is the chain reversible?

(iii) Consider next the biased random walk on $\{0, \ldots, n\}$, i.e. with transition probabilities $P(i, i+1) = 2/3 = 1 - P(i, i-1)$ for all $i \in \{2, \ldots, n-1\}$ and $P(1, 2) = 2/3 = 1 - P(1, 1)$ and $P(n, n-1) = 1/3 = 1 - P(n, n)$. Is this chain reversible?

5. Show that if X is reversible, then for all a, b, c we have

$$\mathbb{E}_a[\tau_b] + \mathbb{E}_b[\tau_c] + \mathbb{E}_c[\tau_a] = \mathbb{E}_a[\tau_c] + \mathbb{E}_c[\tau_b] + \mathbb{E}_b[\tau_a]$$