Cross-diffusion systems for image denoising

A. Araújo
CMUC, Department of Mathematics, University of Coimbra, Portugal.
alma@mat.uc.pt

S. Barbeiro
CMUC, Department of Mathematics, University of Coimbra, Portugal.
silvia@mat.uc.pt

E. Cuesta
Department of Applied Mathematics, University of Valladolid, Spain.
eduardo@mat.uva.es

A. Durán
Department of Applied Mathematics, University of Valladolid, Spain.
angel@mac.uva.es

Abstract

Diffusion processes are commonly used in image processing, as, for example, in noise removal. The main idea is that if one pixel is affected by noise, then the noise should be diffused among the neighboring pixels in order to smooth the region. In this way proper diffusion partial differential equations have been considered to achieve this end.

A foregoing proposal is the use of complex diffusion problems, developed by Gilboa and collaborators, [7], where the image is represented by a complex function which evolves according to some complex diffusion process. The application of nonlinear complex diffusion to image filtering and edge enhancing is discussed in [5, 6, 7]. In particular, complex diffusion models have been successfully applied in medical imaging denoising [4].

The interpretation of complex diffusion as a cross diffusion system motivates the introduction of a more general set of equations and its study as a mathematical model in image processing. As in the complex diffusion case, the idea of dividing the information of the image in two components evolving in a cross way is behind the approach, which may provide a relevant diversity of the resulting models that can be used to adapt their performance to the problem under study. This was shown for the complex diffusion, both in the linear case when studying the role of the imaginary part as smoothed Laplacian of the initial image (the so-called small theta approximation) and in the nonlinear case when some complex shock filter models are proposed, [7].

In this talk we will discuss the use of nonlinear cross-diffusion systems to perform image restoration [2], which follows the previous work for the linear case, [1]. Special attention will be given to the well-posedness of the nonlinear cross-diffusion initial boundary-value problem with Neumann boundary conditions, along with scale-space properties, [3, 8], and long time behaviour of the models. Some of the arguments of [20] for the system under study will be used and generalized here.

Additionally, the performance of the models are estimated by computational means in several examples. The numerical experiments presented seek to be illustrative of the potential of these models to image filtering. Rather than being exhaustive, this numerical study is a proof-of-concept which aims to motivate future research in this direction.

Keywords: Cross-diffusion, Complex diffusion, Image restoration.

References

[1] A. Araújo, S. Barbeiro, E. Cuesta, A. Durán. Cross- diffusion systems for image processing: I. The linear case, Preprint available at http://arxiv.org/abs/1605.02923, 2016.

[2] A. Araújo, S. Barbeiro, E. Cuesta, A. Durán. Cross- diffusion systems for image processing: II. The linear case, Preprint available at https://arxiv.org/abs/1605.03721, 2016.

[3] L. Álvarez, F. Guichard, P.-L. Lions, J.-M. Morel,J.-M., Axioms and fundamental equations for image processing. Arch. Rational Mech. Anal. 123: 199-257, 1993.

[4] H. Salinas, D. Fernández. Comparison of PDE-based nonlinear diffusion approaches for image enhancement and denoising in optical coherence tomography, IEEE Trans. Med. Imaging, 26: 761-771, 2007.

[5] G. Gilboa, Y.Y. Zeevi, N.A. Sochen, N. A, Complex diffusion processes for image filtering. In: Scale-space and Morphology in Computer Vision, 299-307, Springer, Berlin, 2001.

[6] G. Gilboa, N.A. Sochen, Y.Y. Zeevi. Generalized shock filters and complex diffusion. In: Computer Vision- ECCV: 399-413, Springer, Berlin, 2002.

[7] G. Gilboa, N.A. Sochen, Y.Y. Zeevi. Image enhance-ment and denoising by complex diffusion processes, IEEE Trans. Pattern Analysis and Machine Intelligence. 26(8): 1020-1036, 2004.

[8] T. Lindeberg, Scale Space. In: Encyclopedia of Computer Science and Engineering (B. Wah ed.), Vol IV: 2495-2509, John Willey and Sons, Hoboken New Jersey, 2009.