Analysis of a chemotherapy model for brain tumours

J. R. Branco
CMUC & Polytechnic Institute of Coimbra, ISEC, DFM, Coimbra, Portugal.
jrbranco@isec.pt

J. A. Ferreira
CMUC, Department of Mathematics, University of Coimbra, Portugal.
ferreira@mat.uc.pt

P. de Oliveira
CMUC, Department of Mathematics, University of Coimbra, Portugal.
poliveir@mat.uc.pt

G. Pena
CMUC, Department of Mathematics, University of Coimbra, Portugal.
gpena@mat.uc.pt

Abstract

Tumours are the uncontrolled growth of abnormal cells. Gliomas, which are a special case of brain tumours, have a high mortality rate, giving six months to one year of life expectancy once its diagnosed in patients (even undergoing treatment).

The modelling of migration and proliferation of tumour cells has received attention by the scientific community in the last decades. The model proposed by [3, 4], based on the principles followed by [1], considers tumour cells as having two possible phenotypes (or states), proliferative and migratory, allowing cells to transition between both phenotypes. Later extensions of this model, proposed by [2, 7], allowed for the inclusion of the stiffness effect of the white and gray matter on the cell's mobility as well as the geometry of the white and gray matter distribution. Another point that has been receiving some attention is the effect of chemotherapy or radiotherapy in the control of the cell growth. This has been accomplished by introducing an additional term in the partial differential system that removes cells at a rate depending on the concentration of a certain drug (or an amount of radiation, in the case of radiotherapy), [7, 6, 5].

In this talk we analyse properties of the nonlinear mathematical model proposed in [7] to describe the evolution of brain tumour cells under the effect of a chemotherapy drug,

considering the viscoelastic behaviour of the brain. We prove, under suitable regularity assumptions, upper bounds for an energy of the system with respect to the L^2 norm, as well as the stability of the model. Chemotherapy treatment protocols based on stronger assumptions on the drug and cell kinetics are also proposed with the purpose of controlling tumor growth. The qualitative behaviour of the solutions of the system is explored with a suitable numerical method and discussed.

Keywords: Glioma, chemotherapy, treatment protocols, stability analysis, numerical simulation

References

[1] J. D. Murray, *Mathematical Biology,* Springer, 2002.

[2] J. R. Branco, J. A. Ferreira and P. de Oliveira, *Mathematical modelling of efficient protocols to control glioma growth,* Math Biosci. **255** (2014) 83–90.

[3] S. Fedotov and A. Iomin, *Migration and proliferation dichotomy in tumour-cell invasion,* Phys. Rev. Lett. **98**(118110) (2007) 1–4.

[4] S. Fedotov and A. Iomin, *Probabilistic approach to a proliferation and migration dichotomy in tumour cell invasion,* Phys. Rev. E **77**(1031911) (2008) 1–10.

[5] P. Tracqui, G. C. Cruywagen, D. E. Woodward, G. T. Bartoo, J. D. Murray and E. C. Alvord Jr, *A mathematical model of glioma growth: the effect of chemotherapy on spatio-temporal growth,* Cell Proliferat. **28** (1995) 17–31.

[6] E. S. Norris, J. R. King and H. M. Byrne, *Modelling the response of spatially structured tumours to chemotherapy: Drug kinetics,* Mathematical and Computer Modelling **43** (2006) 820–837.

[7] J. R. Branco, J. A. Ferreira, P. de Oliveira and G. Pena, *Chemotherapy for brain tumour: balance between frequency and intensity,* in Proceedings of the 2015 International Conference on Computational and mathematical Methods in Science and Engineering, Cadiz, Spain, July 6-10, 2015, Editor: J. Vigo Aguiar, (2015) 244–253.