A continuum model about Sarcopenia in skeletal muscle tissue

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Abstract

We propose a mathematical model of skeletal muscle tissue with a reduced performance and mass, which is typical of a geriatric syndrome named *sarcopenia* [6]. Sarcopenia is defined as the loss of skeletal muscle mass and strength that occurs with ageing. Although nowadays this syndrome affects more than 50 millions people, there are no compelling tests for its diagnosis and many efforts are made by the medical community to better understand it. Having this in mind, a mathematical model of sarcopenic muscle can be a valuable resource.

Skeletal muscle tissue is a highly-ordered hierarchical structure: muscular fibres are organized in fascicles, containing a concatenation of millions of sarcomers, which are the fundamental unit of the muscle which produces force (i.e. *activation*). Connective tissue, which is essentially isotropic, fills the spaces among the fibres.

We model the tissue as a transversely isotropic and incompressible continuum material. The former assumption is motivated by the alignment of the muscular fibres, while the latter is ensured by the high water content of the tissue. Focusing our attention only on steady properties, we neglect the viscous effects and restrict ourselves to the framework of hyperelasticity. In the model that we propose, there are four constitutive prescriptions: one for the hyperelastic energy when the tissue is not activated (*passive energy*), one for the activation, one for the loss of mass and one for the loss of performance.

As far as the passive part is concerned, we assume an exponential stress response of the material, which is customary in biological tissues. Denoting with $\mathbf{F}$ the deformation gradient and with $\mathbf{C} = \mathbf{F}^T\mathbf{F}$ the right Cauchy-Green tensor, the hyperelastic strain energy density is given by

$$W(\mathbf{C}) = \frac{\mu}{4}\left\{\frac{1}{\alpha}\left[e^{\alpha(I_p-1)} - 1\right] + K_p - 1\right\}, \tag{1}$$

where I_p and K_p are two generalized invariants which account for the isotropic and anisotropic components of the tissue [2, 3, 4].

Coming to the activation, a recent and very promising way to describe it is the so called *active strain* approach [1], where the extra energy produced by the activation mechanism is encoded in a multiplicative decomposition of the deformation gradient in an elastic and an active part:

$$\mathbf{F} = \mathbf{F}_e \mathbf{F}_a.$$

The active strain approach has firstly been applied to the skeletal muscle tissue in [3].

We, then, consider the presence of sarcopenia, which is one of the novelties of our model. The loss of muscle mass is encoded in the model by a percentage parameter g which measures the fraction of muscle fibres which are not active anymore; the loss of performance is represented by a damage parameter d which reduces the active part of the stress by a given percentage. The lack of experimental data on the elastic properties of a sarcopenic muscle tissue does not allow any fitting of the two parameters; however, the proposed model can be numerically implemented using finite element methods and some different scenarios can be studied.

Finally, we present some results obtained using FEniCS, an open source collection of Python libraries, showing that the experimental results of [5] on the passive and active stress-strain healthy curves, obtained *in vivo* from a tetanized tibialis anterior of a rat, can be well reproduced by our model in the healthy case. Further, the behavior of the tissue when g and d increase is discussed.

Keywords: Hyperelasticity, activation, ageing, active strain, biomechanics.

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