Navier-Stokes flows around moving obstacles

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Abstract

Consider a rigid body $\mathcal{S} \subset \mathbb{R}^3$ moving through a viscous incompressible fluid which fills the exterior domain $\Omega := \mathbb{R}^3 \setminus \mathcal{S}$ and the steady state regime of the system body-fluid, as seen by an observer attached to $\mathcal{S}$. The equations modeling this mechanical system are

$$-\nabla \cdot \sigma(v,p) + \varrho(v - V) \cdot \nabla v + \varrho\omega \times v = \varrho f \quad \text{in } \Omega \tag{1}$$

$$\nabla \cdot v = 0 \quad \text{in } \Omega \tag{2}$$

$$v = V + v_* \quad \text{on } \partial\Omega \tag{3}$$

$$\lim_{|x|\to\infty} v(x) = 0 \tag{4}$$

where the quantities $v = v(x)$ and $p = p(x)$ represent, respectively, the unknown velocity field and the pressure of the liquid, $V(x) = \xi + \omega \times x$ represents the velocity of $\mathcal{S}$, v_* is a distribution of velocities which describes a possible motion of the surface of $\mathcal{S}$ and f is an external force acting on the fluid. The positive constant ϱ is the density of the fluid and $\sigma(v,p)$ is the Cauchy stress tensor, $\sigma(v,p) := 2\nu D(v) - pI_3$, ν being the viscosity of the fluid, I_3 the 3×3 identity matrix and $D(v) := \frac{1}{2}\left((\nabla v) + (\nabla v)^\top\right)$.

During the last decades, many authors have been interested in the study of equations (1)-(4), since the effect of the rotation of the solid brings additional challenges to the exterior Navier-Stokes problem. Along with classical mathematical problems such as existence, uniqueness and regularity of solutions to system (1)-(4), it is important to have a rigorous justification of *energy equation*

$$2\nu \int_\Omega |D(v)|^2 \, dx - \int_{\partial\Omega} \big(\sigma(v,p)n\big) \cdot (V + v_*) \, d\gamma = \varrho \int_\Omega f \cdot v \; dx - \frac{\varrho}{2} \int_{\partial\Omega} (v_* \cdot n)|V + v_*|^2 \, d\gamma, \tag{5}$$

and of the observed anisotropic structure of the flow, characterized by the formation of a *wake region* behind the solid whose "width" apparently depends on the angle between ω and ξ.

The generalized Oseen fundamental solution has a crucial role in the derivation of precise information about the asymptotic behavior of v and p at infinity, which, in turn, for small flows, allows to justify (5) and the non-uniform behavior of v when $\omega = 0$ and $\xi \neq 0$ or when $\xi \cdot \omega \neq 0$: the velocity decays faster outside a infinite paraboloidal region behind the body, representative of the wake. The decay structure of v and p is also important when studying the stability and attainability of steady motions, in flow control in exterior domains and for finding appropriate artificial boundary conditions needed for numerical purposes.

This talk is based on joint works with G. P. Galdi (Univ. Pittburgh, USA), T. Takahashi (Inria Nancy Grand-Est, France) and T. Hishida (Nagoya University, Japan) and follows essentially the references below.

Keywords: Navier-Stokes equations, exterior domain, rotating body, generalized Oseen fundamental solution, asymptotic behavior.

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