Numerical simulations of two-dimensional neural fields with applications to working memory

Pedro M. Lima

CENTRO DE MATEMÁTICA COMPUTACIONAL E ESTOCÁSTICA
INSTITUTO SUPERIOR TÉCNICO
UNIVERSIDADE DE LISBOA
PORTUGAL

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OUTLINE OF THE LECTURE

1. **Introduction**
2. **Mathematical Formulation and Numerical Algorithms**
3. **Numerical Examples**
4. **Conclusions and ongoig research**

INTRODUCTION: DYNAMICAL NEURAL FIELDS

Dynamical Neural Fields (DNF) were introduced in the 1970 as simplified mathematical models of pattern formation in neural tissue in which the interaction of billions of neurons is treated as a continuum.

Advantage of DNF:
Explain the existence of self-sustained neuronal activity patterns which are linked to higher cognitive functions such as decision making, memory, prediction or learning.

INTRODUCTION: APPLICATIONS IN ROBOTICS

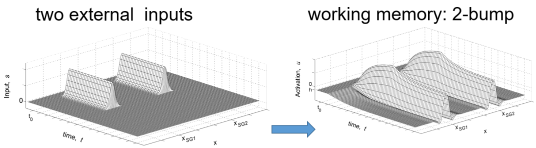
Applications in Robotics:

- Neurodynamics approach to cognitive robotics
- Navigation in environments cluttered with obstacles
- Natural human-robot interactions

INTRODUCTION: WORKING MEMORY

Working Memory is the capacity of neurons to transiently hold sensory information to guide forthcoming action.

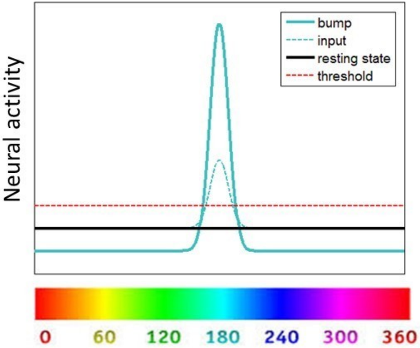
Persistent neural activity observed in many brain areas is thought to represent a neural mechanism underlying working memory.
DNF models support one or more spatially localized activity patterns - bumps- that are initially triggered by sufficiently strong external stimuli and remain self-sustained after stimulus removal.



INTRODUCTION: WORKING MEMORY

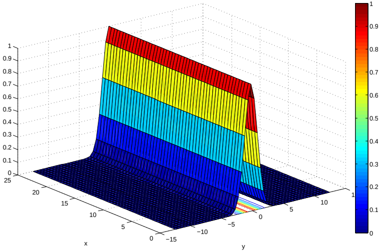
In typical 1-D DNF working memory applications, the field dimension corresponds to continuous stimulus parameters such as color, direction or tone pitch.

So, if for example the neurons in the field encode color, a transient color input may switch between a homogeneous resting state and a stable bump state representing the memory of the specific color event.

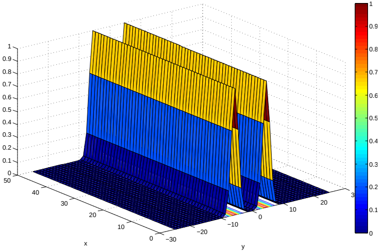


2D NEURAL FIELDS

Populations of cortical neurons may encode in their firing pattern simultaneously the nature and the timing (or temporal order) of sequential stimulus events.
The nature of the event (for example, color) is coded in an input with a certain coordinate y, which extends in the x coordinate.



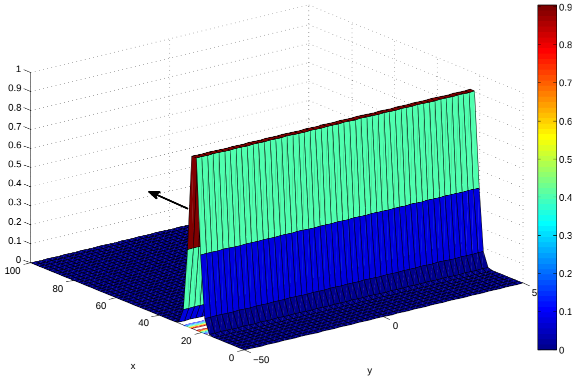
stimulus of 1 color



stimulus of 2 colors

TRAVELING WAVE

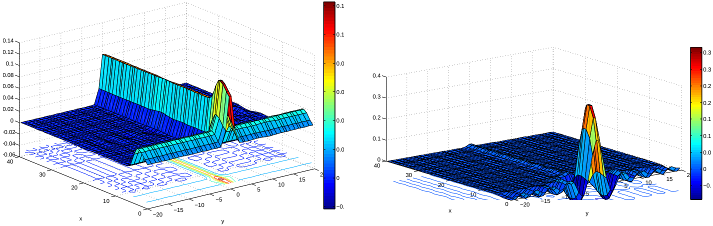
The second input to the field is a traveling wave in form of a ridge which extends in y direction and propagates in the direction of x with elapsed time t since sequence onset at $t = 0$.



$$I_0(x, y, t) = \exp(-\gamma_0 (x - v\, t)^2)$$

GENERATION OF SELF-SUSTAINED ACTIVITY

traveling wave + localized input $\Rightarrow$ self-sustained bump



a *b*

(a) Combination of two inputs (b) Example of a stable bump solution which remains after all the inputs are switched off. The coordinates of this bump represent the nature and the time of the event.

NEURAL FIELD EQUATION

We consider the Neural Field Equation in the form

$$c\frac{\partial}{\partial t}V(x,y,t) = I(x,y,t) - V(x,y,t) +$$

$$\int_\Omega K(\|(x,y)-(x',y')\|_2)S(V(x',y',t))dx'dy', \tag{1}$$

$$t \in [0,T], (x,y) \in \Omega \subset \boldsymbol{R}^2,$$

where $V(x,y,t)$ - represents the potential at (x,y) and instant t.
$\Omega = [0,2L] \times [-L,L]$. The connectivity kernel K is of oscillating type:

$$K(r) = A\exp(-kr)\left(k\sin(a_1 r) + \cos(a_1 r)\right), \tag{2}$$

In our numerical experiments, $A = 0.02, k = 0.8, a_1 = 1$.
The firing rate S is the Heaviside function.:
$S(V) = 0$, if $V < b$; $S(V) = 1$, if $V \geq b = 0.1$.

NUMERICAL ALGORITHM

- Time Discretization: second order implicit scheme;
- Space Discretization: Gaussian quadratures; 4 points at each subinterval.
- Improvement of Efficiency: Interpolation at Chebyshev points.

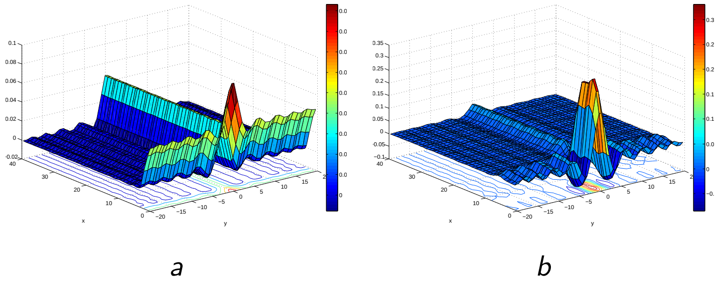
This numerical method has been introduced before ; its stability and convergence have been proved.

NUMERICAL EXAMPLES

Example 1. External input:
if $t \in [0, 1.5]$, $I(t)$ = travelling wave I_0 + localized signal I_1 (with one bump);
if $t > 1.5$, $I(t) \equiv 0$.

$$I_0(x, y, t) = \exp(-\gamma_0(x - v\,t)^2), \qquad I_1(x, y, t) = \alpha_1 \exp(-\gamma_1(x - C_1)^2);$$

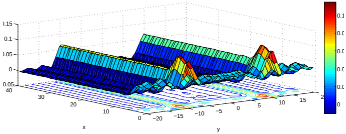
$v, \gamma_i, \alpha_1 > 0$. The domain of discretization is $[0, 40] \times [-20, 20]$;



a) solution at time $t = 0.5$;
b) solution at time $t = 2.5$.

NUMERICAL EXAMPLES

Example 2. External input:
if $t \in [0,1]$, $I(t) =$ traveling wave I_0 + two colors $I_1 + I_2$;
if $t > 1$, $I(t) \equiv 0$. The domain of discretization is $[0,40] \times [-20,20]$;



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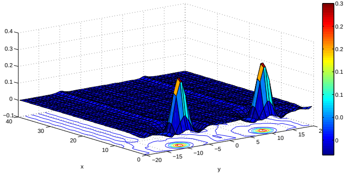
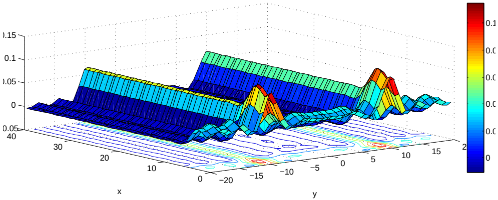


b

a) solution at time $t = 1$;
b) solution at time $t = 5$.

NUMERICAL EXAMPLES

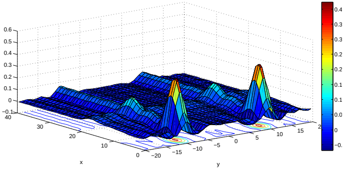
Example 3. External input:
if $t \in [0,1]$, $I(t) =$ traveling wave I_0 + two colors $I_1 + I_2$;
if $t \in [1,3]$, $I(t) \equiv I_0$;
if $t \in [3,4]$, $I(t) =$ traveling wave I_0 + two colors $I_1 + I_2$;
if $t > 4$, $I(t) \equiv 0$.



Solution at time $t = 1$; here the output field contains only the representation of the first series of signals.

NUMERICAL EXAMPLES

Example 3 (continued)



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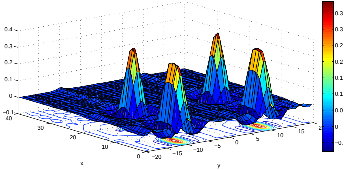
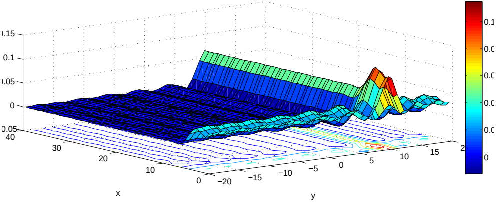


b

a) Surface graphs of the solution at time $t = 4$; at this moment we can see also a representation of the second series of signals;
b) Surface graphs of the solution at time $t = 7$; here we can see the stable four-bump field which remains after all the inputs are switched off.

NUMERICAL EXAMPLES

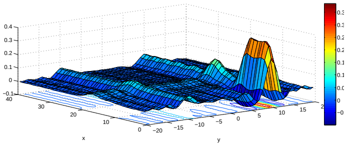
Example 4. External input:
if $t \in [0, 1.5]$, $I(t) =$ traveling wave I_0 + one color I_1;
if $t \in [1.5, 3]$, $I(t) \equiv I_0$;
if $t \in [3, 4.5]$, $I(t) =$ traveling wave I_0 + two colors $I_1 + I_2$;
if $t > 4.5$, $I(t) \equiv 0$.



Solution at time $t = 1$; here the output field contains only the representation of the first signal (one color).

NUMERICAL EXAMPLES

Example 4 (continued)



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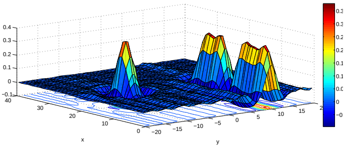

b

a) Surface graphs of the solution at time $t = 4$; at this moment we can see also a representation of the second series of signals.
b) Surface graphs of the solution at time $t = 7$; here we can see the stable three-bump field which remains after all the inputs are switched off.

CONCLUSIONS AND ONGOING RESEARCH

- We have described a two-dimensional neural field model which explains how a population of cortical neurons may encode in its self-sustained firing patternsimultaneously the nature and time of sequential stimulus events.
- The postulated wave mechanism explains how a nervous system lacking specific sensors for temporal perception may develop neurons that respond to specific interval durations.
- The numerical results presented support the conjecture that if the external input has appropriate intensity and duration, and if the connection kernel is of the oscillatory type described here, The neural activity can generate stable multibump solutions which contain the information carried by the external signals.

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