Riemannian Geometry
MMAC
Fall Semester, 2021/22

Professor: Pedro M. Girão

Course Description

- Differentiable Manifolds: tangent space; differentiable maps; immersions and embeddings; vector fields; Lie brackets; Lie groups; differential forms. Riemannian Manifolds: isometries; affine connections, Levi-Civita connection; geodesics, minimizing properties of geodesics; Hopf-Rinow theorem. Curvature: curvature tensor, sectional curvature, Ricci tensor, scalar curvature; connection and curvature forms, Cartan's structure equations; isometric immersions of surfaces in euclidean space, Gauss map, mean and Gaussian curvature; Gauss Theorem; first and second fundamental forms. Applications: index of a vector field at a singularity; the Euler-Poincaré characteristic; the theorem of Gauss-Bonnet.

Readings

- *An Introduction to Riemannian Geometry*, L. Godinho and J. Natário, Springer, 2014 (Main text).
- *An Introduction to Differentiable Manifolds and Riemannian Geometry*, W. Boothby, Academic Press, 2003.
- *Notes on Differential Geometry*, N. Hicks, D. Van Nostrand Company, 1965.
- *Differential Geometry*, Will J. Merry, Lecture Notes.
- *Riemannian Geometry*, M. Perdigão do Carmo, Birkhäuser, 1992.
- *Differential Forms and Applications*, M. Perdigão do Carmo, Springer, 1994.

Evaluation

- Homeworks are due two weeks after they are assigned. There will be two exams, E1 (July 6, 2022, at 1pm) and E2 (July 18, 2022, at 3:30 pm) The final grade is $\max\left\{\frac{H+E1}{2}, \frac{H+E2}{2}, E1, E2\right\}$. Final grades greater than or equal to 19 have to be confirmed in an oral exam.