

# Mathematical Relativity

## PhD Program in Mathematics

### Spring Semester, 2022/23

**Professor:** Pedro Girão

#### Course Description

1. **Examples.** Minkowski spacetime; Schwarzschild solution; Einstein, de Sitter, anti-de Sitter and Friedmann-Lemaître-Robertson-Walker universes; matching and Oppenheimer-Snyder collapse; Penrose diagrams.
2. **Causality.** time orientability, chronological and causal past and future, domains of dependency; chronological, stably causal and globally hyperbolic spacetimes.
3. **Singularities.** Jacobi equation, conjugate points; energy conditions; existence of maximizing geodesics; Hawking and Penrose theorems.
4. **Cauchy Problem.** Wave equation; Cauchy problem with restrictions; Gauss-Coddazzi relations and 3+1 decomposition of the Einstein equations; Choquet-Bruhat theorem; restriction equations for the initial data.
5. **Positive mass theorem.** Komar mass; Einstein-Hilbert action; Lagrangian and Hamiltonian formulation of the Einstein equations; mass of an asymptotically flat Riemannian manifold; Positive mass theorem; Penrose's inequality.
6. **Black holes.** The Kerr solution; Killing horizons and the zeroth law; Smarr's formula and the first law; Second law; Hawking radiation and black hole thermodynamics.

#### Office hours

Tuesdays and Thursdays, after class.

#### Evaluation

Homework is due two weeks after it is assigned and is worth 50% of the final grade. There will be two exams. If a student decides to attend both of them, the best of the two grades will be considered and it is worth 50% of the final grade. To be approved a student must have an exam grade of 8 or more. Final grades of 19 or 20 must be confirmed in an oral exam.