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INSTITUTO SUPERIOR TÉCNICO

DIFFERENTIAL GEOMETRY

1st *Semester - 2025/26 (MMAC, PDM)*

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Program

- Differentiable manifolds: definition and examples; tangent and cotangent spaces; Whitney's theorem.
- Vector fields and distributions: vector fields, flows, diffeomorphisms; Lie bracket and Lie derivative; involute distributions and Frobenius's theorem.
- Tensor algebra and tensor fields: multilinear algebra; covariant and contravariant tensors; contractions and Lie derivative.
- Differential forms and DeRham cohomology: exterior algebras and differential forms; exterior differential; differential ideals and Frobenius's theorem; integration and DeRham cohomology.
- Connections: affine connections; curvature and Bianchi identities; parallel transport and geodesics.
- Riemannian manifolds: metrics on manifolds; Levi-Civita connection; curvature tensor; sectional curvature, Ricci curvature and scalar curvature.
- Global differential geometry: curvature and topology relations; fiber bundles and characteristic classes.

There will be plenty of examples illustrating the whole program. Namely, we will discuss Riemannian surfaces, Lie groups, homogeneous spaces and toric manifolds.

Bibliography

- R. L. Fernandes, *Differential Geometry*, 2021.
- M. Carmo, *Geometria Riemanniana*, IMPA, 1988
- J. Jost, *Riemannian Geometry and Geometric Analysis, 2nd ed.*, Springer-Verlag, 1998.
- M. Spivak, *A comprehensive introduction to Differential Geometry, Vols. I-V*, Publish or Perish, 1970-79.
- J.M. Lee, *Riemannian Manifolds: an introduction to curvature*, Springer-Verlag, 1997.

Homeworks and Exam

There are 6 homework assignments during the semester and one final exam in January. The final grade is the weighted average of the grade of the final exam (weight 70%) and of the average of the top five grades of the homework assignments (weight 30%).