Geometric Mechanics

Homework 7

Due on November 11

1. Let $(M, \langle\cdot,\cdot\rangle)$ be a Riemannian manifold and $p, q \in M$. A curve of minimal length connecting p to q, with nonvanishing derivative, must be a critical point of the action determined by the Lagrangian $L : TM \setminus Z \to \mathbb{R}$ given by

$$L(v) = \langle v, v\rangle^{\frac{1}{2}}$$

($Z = \{0 \in T_pM : p \in M\}$ is the zero section).

   (a) Show that such a curve is a reparameterized geodesic. (**Hint:** Write $L(v) = (2K(v))^{\frac{1}{2}}$).

   (b) Compute the Hamiltonian function $H : TM \setminus Z \to \mathbb{R}$.

2. Consider the action of $SO(3)$ on itself by left multiplication.

   (a) Show that the infinitesimal action of $B \in \mathfrak{so}(3)$ is the vector field X^B given by

$$\left(X^B\right)_S = BS.$$

   (b) Use the Noether Theorem to show that the angular momentum $p = SI\Omega$ of the free rigid body is constant.